Regular Theories and their Monads

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(joint work with Stanisław Szawiel)

University of Warsaw

Workshop on Category Theory
In honour of George Janelidze,
on the occasion of his 60th birthday
Coimbra, July 10, 2012

Categories of Equational Theories

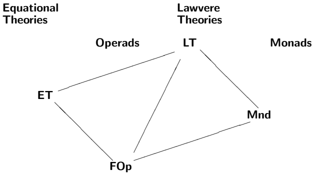
Equational Theories

Lawvere Theories

Operads

Monads

Categories of Equational Theories



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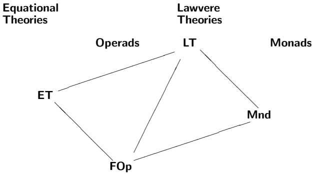

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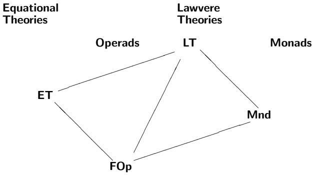

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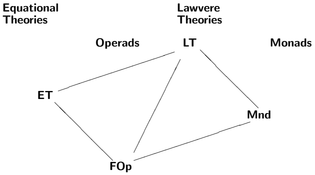

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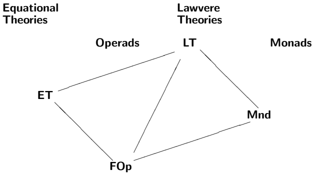

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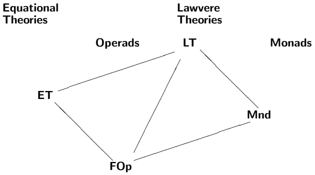

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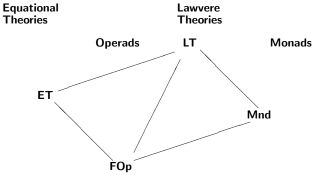

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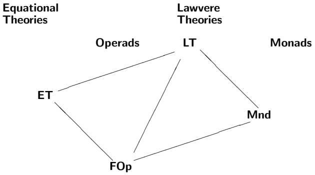

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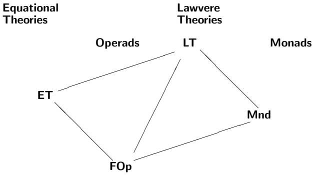

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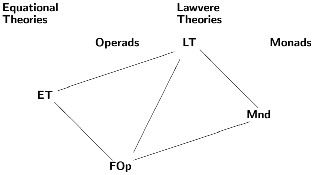

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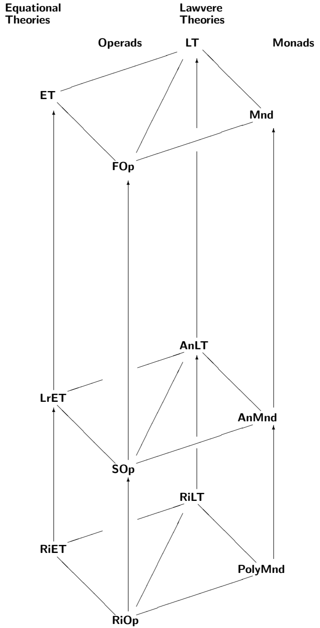

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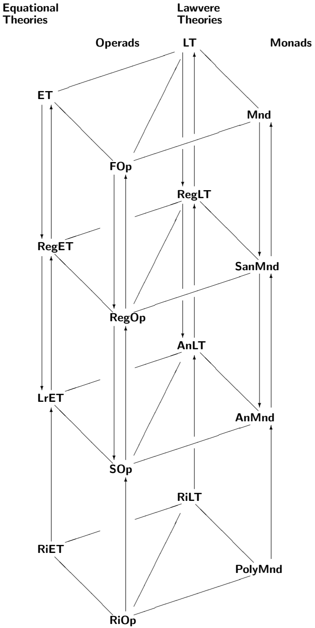

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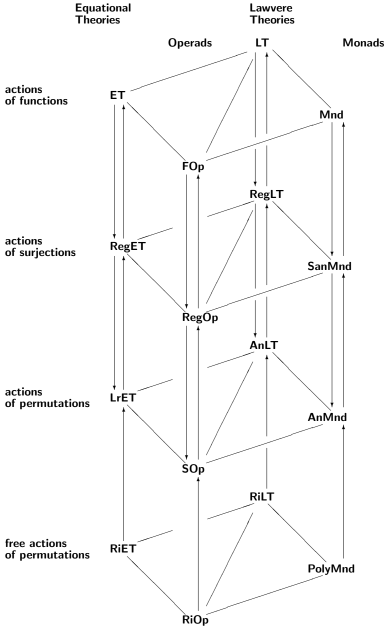

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Regular theory

A an equational theory T is *regular* iff it has a set of regular axioms.

Regular interpretation

An interpretation of equational theories $I : T \longrightarrow T'$ is *regular* iff it interprets n-ary symbols f in T as regular terms in contexts $t : \vec{x}^n$ in T'.

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Examples of regular theories

- The theory of sup-lattices: two operations $\vee$ and $\perp$, of arity 2 and 0, respectively, and equations

$$x_1 \vee (x_2 \vee x_3) = (x_1 \vee x_2) \vee x_3, \quad x_1 \vee \perp = x_1 = \perp \vee x_1,$$

$$x_1 \vee x_2 = x_2 \vee x_1, \quad x_1 \vee x_1 = x_1$$

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- Monoids, monoids with involutions, abelian monoids.

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- Monoids, monoids with involutions, abelian monoids.
- Groups, rings, modules ARE NOT!

Regular operads and Semi-analytic monads
semi-analytic functors

- $i : \mathbb{S} \to \mathbb{F}$ is an inclusion of a subcategory with the same objects whose morphisms are surjections

$$\begin{array}{ccc} Set^{\mathbb{F}} & \xrightarrow{\;Lan_{\iota_{\mathbb{F}}}\ \simeq\;} & \mathbf{End} \\ {\scriptstyle Lan_i}\uparrow & & \uparrow \\ Set^{\mathbb{S}} & \xrightarrow[\;Lan_{\iota_{\mathbb{S}}}\ \simeq\;]{} & \mathbf{San} \end{array}$$

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- the monoids in **San** is the category of semi-analytic monads **SanMnd**.

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- $A : \mathbb{S} \to Set$ functor then on A_n we have a left action of S_n

$$S_n \times A_n \longrightarrow A_n$$

$$\langle \tau, a \rangle \mapsto A(\tau)(a)$$

- Dividing $\begin{bmatrix} Y \\ n \end{bmatrix} \times A_n$ by the relation

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- ... and whole semi-analytic series

$$\hat{A}(Y) = \sum_{n \in \omega} \left[\begin{array}{c} Y \\ n \end{array}\right] \otimes_n A_n$$

which IS functorial in Y!

- $f : X \to Y$ -function, $[\vec{x}, a]$ an element of $\left[\begin{array}{c} X \\ n \end{array} \right] \otimes_n A_n$

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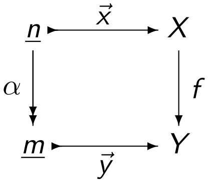

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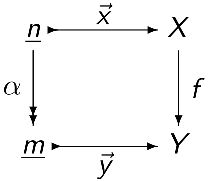

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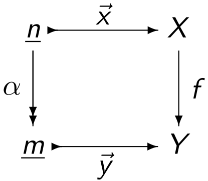

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$$\begin{array}{ccc} \underline{n} & \stackrel{\vec{x}}{\longrightarrow} & X \\ {\scriptstyle \alpha}\downarrow & & \downarrow{\scriptstyle f} \\ \underline{m} & \underset{\vec{y}}{\longrightarrow} & Y \end{array}$$

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  which is an element in $\left[\begin{array}{c} Y \\ m \end{array} \right] \otimes_m A_m$.
- Thus we have defined $\hat{A}(f) : \hat{A}(X) \longrightarrow \hat{A}(Y)$

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- Thus we have a functor

$$(\hat{-}) : Set^{\mathbb{S}} \longrightarrow \mathbf{End}$$

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- The functor

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associating to a set X the set of subsets of X with at most n-elements is not analytic, if $n > 2$, as it can be easily seen that it does not preserve weak pullbacks. However, it preserves pullbacks along monos and hence it is semi-analytic.

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- The functor part of any monad on *Set* that comes from a regular equational theory is semi-analytic.

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- the category **RegLT** of regular Lawvere theories monads.

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Simple automorphisms

We say that Lawvere theory T has *simple automorphisms* iff ρ_n is a bijection, for $n \in \omega$.

Projection morphisms

The class of *projections* in T is the closure under isomorphism of the image under π of all monomorphisms in $\mathbb{F}$.

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A morphism in T is *regular* iff it is right orthogonal to all projections.

Regular Lawvere theory

Lawvere theory T is *regular* iff

- T has simple automorphisms;
- projections and regular morphisms form a factorization system in T.

Interpretations of Regular Lawvere theories

A *regular interpretation* of Lawvere theories $I : T \to T'$ is an interpretation of Lawvere theories that preserves regular morphisms.

The end

Happy Birthday George!