TYPES as KAN COMPLEXES

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Constructive type theory was introduced by Martin-Loef as a foundation for constructive mathematics. Its impredicative version is implemented within the Coq system which allows to construct proofs interactively as elements of a given type representing a proposition. In this sense it provides a synthesis of (constructive) logic and functional programming.

In order to make type checking decidable one has to distinguish between "judgemental" and "propositional" equality. The former corresponds to "conversion" and is decidable whereas the latter is (closer to) mathematical equality which is highly non-decidable.

This distinction between two notions of equality makes life a bit cumbersome. But it allows one to interpret propositional equality as being isomorphic as in the Groupoid Model of Hofmann and Streicher from 1993. Already there it was suggested that actually types should not be interpretad as groupoids but as "weak higher dimensional groupoids". The most accessible notion of weak higher dimensional groupoid is provided by Kan complexes within the topos SSet of simplicial sets as suggested independently by Streicher and Voevodsky in 2006.

Recently Voevodsky has shown that this model validates his "Equivalence Axiom" stating that types are equal iff there is a "weak equivalence" between them. Here "weakly equivalent" means that there is a map between the types which is surjective when this condition is expressed in terms of propositional equality. The intention is that extending Coq this way allows one to consider isomorphic types as equal as it is common in category theory.

Our lectures are organized into 3 parts

1. Type Theory and its Categorical Models

2. Basics of Simplicial Homotopy Theory

3. Interpreting Type Theory in SSet and Voevodsky's Equivalence Axiom