Naturalna dedukcja

$$\frac{\Gamma \vdash \varphi \quad \Gamma \vdash \psi}{\Gamma \vdash \varphi \,\&\, \psi}\ (+\&) \qquad \frac{\Gamma \vdash \varphi \,\&\, \psi}{\Gamma \vdash \varphi}(-\&)\frac{\Gamma \vdash \varphi \,\&\, \psi}{\Gamma \vdash \psi}$$

$$\frac{\Gamma \vdash \varphi \quad \Delta \vdash \psi}{\Gamma, \Delta \vdash \varphi \otimes \psi}\ (+\otimes) \qquad \frac{\Gamma, \varphi, \psi \vdash \sigma \quad \Delta \vdash \varphi \otimes \psi}{\Gamma, \Delta \vdash \sigma}\ (-\otimes)$$

$$\frac{\Gamma \vdash \varphi}{\Gamma \vdash \varphi \oplus \psi}\ (+\oplus)\ \frac{\Gamma \vdash \psi}{\Gamma \vdash \varphi \oplus \psi} \qquad \frac{\Gamma, \varphi \vdash \sigma \quad \Gamma, \psi \vdash \sigma \quad \Delta \vdash \varphi \oplus \psi}{\Gamma, \Delta \vdash \sigma}\ (-\oplus)$$

$$\frac{\Gamma, \varphi \vdash \psi}{\Gamma \vdash \varphi \multimap \psi}\ (+\multimap) \qquad \frac{\Gamma \vdash \varphi \multimap \psi \quad \Delta \vdash \varphi}{\Gamma, \Delta \vdash \psi}(-\multimap)$$

$$\vdash 1\ (+1) \qquad \frac{\Gamma \vdash \sigma \quad \Delta \vdash 1}{\Gamma, \Delta \vdash \sigma}\ (-1) \qquad \Gamma \vdash \top\ (+\top) \qquad \frac{\Delta \vdash 0}{\Gamma, \Delta \vdash \varphi}\ (-0)$$

$$\frac{\Gamma \vdash \sigma \quad \Delta \vdash\, !\varphi}{\Gamma, \Delta \vdash \sigma}\ (-\mathrm{O}!) \qquad \frac{\Gamma, !\varphi, !\varphi \vdash \sigma \quad \Delta \vdash\, !\varphi}{\Gamma, \Delta \vdash \sigma}\ (-\mathrm{S}!)$$

$$\frac{\Delta_1 \vdash\, !\gamma_1 \cdots \Delta_n \vdash\, !\gamma_n \quad\ !\Gamma \vdash \varphi}{\Delta_1, \ldots, \Delta_n \vdash\, !\varphi}\ (+!) \qquad \frac{\Gamma \vdash\, !\varphi}{\Gamma \vdash \varphi}\ (-\mathrm{L}!)$$

Termy dla naturalnej dedukcji

$$\frac{\Gamma \vdash M : \varphi \quad \Gamma \vdash N : \psi}{\Gamma \vdash \langle M, N \rangle : \varphi \,\&\, \psi}\ (+\&) \qquad \frac{\Gamma \vdash M : \varphi \,\&\, \psi}{\Gamma \vdash \pi_1(M) : \varphi}(-\&)\frac{\Gamma \vdash M : \varphi \,\&\, \psi}{\Gamma \vdash \pi_2(M) : \psi}$$

$$\frac{\Gamma \vdash M : \varphi \quad \Delta \vdash N : \psi}{\Gamma, \Delta \vdash M \otimes N : \varphi \otimes \psi}\ (+\otimes) \qquad \frac{\Gamma, x : \varphi, y : \psi \vdash M : \sigma \quad \Delta \vdash N : \varphi \otimes \psi}{\Gamma, \Delta \vdash \mathbf{let}\ x \otimes y = N\ \mathbf{in}\ M : \sigma}\ (-\otimes)$$

$$\frac{\Gamma \vdash M : \varphi}{\Gamma \vdash \mathbf{inl}_{\varphi \oplus \psi}(M) : \varphi \oplus \psi}\ (+\oplus)\ \frac{\Gamma \vdash M : \psi}{\Gamma \vdash \mathbf{inr}_{\varphi \oplus \psi}(M) : \varphi \oplus \psi}$$

$$\frac{\Gamma, x : \varphi \vdash M : \sigma \quad \Gamma, y : \psi \vdash N : \sigma \quad \Delta \vdash P : \varphi \oplus \psi}{\Gamma, \Delta \vdash \mathbf{case}\ P\ \mathbf{of}\ [x]M, [y]N : \sigma}\ (-\oplus)$$

$$\frac{\Gamma, x : \varphi \vdash M : \psi}{\Gamma \vdash \lambda x.\, M : \varphi \multimap \psi}\ (+\multimap) \qquad \frac{\Gamma \vdash M : \varphi \multimap \psi \quad \Delta \vdash N : \varphi}{\Gamma, \Delta \vdash MN : \psi}(-\multimap)$$

$$\vdash \bullet : 1\ (+1) \qquad \frac{\Gamma \vdash M : \sigma \quad \Delta \vdash P : 1}{\Gamma, \Delta \vdash \mathbf{let}\ \bullet = P\ \mathbf{in}\ M : \sigma}\ (-1)$$

$$\vec{x} : \Gamma \vdash \mathit{Top}(\vec{x}) : \top\ (+\top) \qquad \frac{\vec{y} : \Delta \vdash M : 0}{\vec{x} : \Gamma, \vec{y} : \Delta \vdash \varepsilon_\varphi(\vec{x}, M) : \varphi}\ (-0)$$

$$\frac{\Gamma \vdash M : \sigma \quad \Delta \vdash N : !\varphi}{\Gamma, \Delta \vdash \mathbf{discard}\ N\ \mathbf{before}\ M : \sigma}\ (-\mathrm{O}!) \qquad \frac{\Gamma, x : !\varphi, y : !\varphi \vdash M : \sigma \quad \Delta \vdash N : !\varphi}{\Gamma, \Delta \vdash \mathbf{share}\ N\ \mathbf{as}\ x, y\ \mathbf{in}\ M : \sigma}\ (-\mathrm{S}!)$$

$$\frac{\Delta_1 \vdash P_1 : !\gamma_1 \cdots \Delta_n \vdash P_n : !\gamma_n \quad \vec{x} : !\Gamma \vdash M : \varphi}{\Delta_1, \ldots, \Delta_n \vdash \mathbf{promote}\ P_1, \ldots, P_n\ \mathbf{for}\ \vec{x}\ \mathbf{in}\ M : !\varphi}\ (+!) \qquad \frac{\Gamma \vdash N : !\varphi}{\Gamma \vdash \mathbf{consume}(N) : \varphi}\ (-\mathrm{L}!)$$

Beta redukcja

$(\lambda x.\, M)N \quad \to \quad M[N/x];$ $\qquad$ $\pi_i(< M_1, M_2 >) \quad \to \quad M_i;$

$\mathbf{let}\ \bullet = \bullet\ \mathbf{in}\ M \quad \to \quad M;$ $\qquad$ $\mathbf{let}\ x \otimes y = P \otimes Q\ \mathbf{in}\ M \quad \to \quad M[P/x, Q/y];$

$\mathbf{case}\ \mathbf{inl}(P)\ \mathbf{of}\ [x]M, [y]N \quad \to \quad M[P/x];$ $\qquad$ $\mathbf{case}\ \mathbf{inr}(Q)\ \mathbf{of}\ [x]M, [y]N \quad \to \quad N[Q/y];$

$\mathbf{discard}\ (\mathbf{promote}\ \vec{P}\ \mathbf{for}\ \vec{x}\ \mathbf{in}\ N)\ \mathbf{before}\ M \quad \to \quad \mathbf{discard}\ \vec{P}\ \mathbf{before}\ M;$

$\mathbf{share}\ (\mathbf{promote}\ \vec{P}\ \mathbf{for}\ \vec{x}\ \mathbf{in}\ M)\ \mathbf{as}\ y, z\ \mathbf{in}\ N \quad \to$

$\quad \mathbf{share}\ \vec{P}\ \mathbf{as}\ \vec{x}_1, \vec{x}_2\ \mathbf{in}\ N[(\mathbf{promote}\ \vec{x}_1\ \mathbf{for}\ \vec{x}\ \mathbf{in}\ M)/y, (\mathbf{promote}\ \vec{x}_2\ \mathbf{for}\ \vec{x}\ \mathbf{in}\ M)/z];$

$\mathbf{consume}(\mathbf{promote}\ \vec{P}\ \mathbf{for}\ \vec{x}\ \mathbf{in}\ M) \quad \to \quad M[\vec{P}/\vec{x}];$

Ewaluacja termów zamkniętych wg Abramsky'ego

$$\bullet \Downarrow \bullet \qquad \lambda x.\, M \Downarrow \lambda x.\, M \qquad \langle\, M, N\,\rangle \Downarrow \langle\, M, N\,\rangle \qquad !\, M \Downarrow !\, M$$

$$\frac{M \Downarrow \langle\, P, Q\,\rangle \quad P \Downarrow c}{\pi_1(M) \Downarrow c} \qquad \frac{M \Downarrow \langle\, P, Q\,\rangle \quad Q \Downarrow d}{\pi_2(M) \Downarrow d}$$

$$\frac{M \Downarrow c \quad N \Downarrow d}{M \otimes N \Downarrow c \otimes d} \qquad \frac{N \Downarrow c \otimes d \quad M[c/x, d/y] \Downarrow e}{\mathbf{let}\ x \otimes y = N\ \mathbf{in}\ M \Downarrow e} \qquad \frac{N \Downarrow \bullet \quad M \Downarrow e}{\mathbf{let}\ \bullet = N\ \mathbf{in}\ M \Downarrow e}$$

$$\frac{M \Downarrow \lambda x.\, P \quad N \Downarrow c \quad P[c/x] \Downarrow d}{MN \Downarrow d} \qquad \frac{M \Downarrow c}{\mathbf{inl}(M) \Downarrow \mathbf{inl}(c)} \qquad \frac{M \Downarrow c}{\mathbf{inr}(M) \Downarrow \mathbf{inr}(c)}$$

$$\frac{M \Downarrow \mathbf{inl}(c) \quad P[c/x] \Downarrow d}{\mathbf{case}\ M\ \mathbf{of}\ [x]P, [y]Q \Downarrow d} \qquad \frac{M \Downarrow \mathbf{inr}(c) \quad Q[c/y] \Downarrow d}{\mathbf{case}\ M\ \mathbf{of}\ [x]P, [y]Q \Downarrow d}$$

$$\frac{N \Downarrow !\, P \quad M \Downarrow c}{\mathbf{discard}\ N\ \mathbf{before}\ M \Downarrow c} \qquad \frac{N \Downarrow !\, P \quad M[!\, P/x, !\, P/y] \Downarrow c}{\mathbf{share}\ N\ \mathbf{as}\ x, y\ \mathbf{in}\ M \Downarrow c} \qquad \frac{N \Downarrow !\, P \quad P \Downarrow c}{\mathbf{consume}(N) \Downarrow c}$$