Algorithmic Trends

Homework 6

Marek Cygan and Piotr Sankowski
May 29, 2014
The homework is due on June 18, 2014

Problem 1. Given a rooted tree, the edges of which have integral weights show how to perform an $O(n)$ preprocessing, allowing $O(1)$ time queries $sumpath(a, b)$, which gives the sum of all the edges on the unique path between vertices a and b.

Problem 2. Analyse the running time of operations INSERT, DELETE, MIN, PREDECESSOR in the van Embe Boas trees, assuming that instead of partitioning the range into $\sqrt{u}$ parts of size $\sqrt{u}$ each, we partition the range into $u^{1/k}$ parts of size $u^{1-1/k}$ each, for a fixed integer $k \geqslant 3$.

Problem 3. Given a binary array of length n, the $select(k)$ operation returns the position of the k-th one in the array (you can assume that the query will only be invoked for a value k such that there are at most k ones in the array).

Show that it is possible to perform $O(n)$-time preprocessing, with only $o(n)$ space usage (additional to the array), such that later each $select(k)$ operation can be performed in $O(1)$ time.