A Generalised Theory of Proportionality in Collective Decision Making

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UNIVERSITAS VARSOVIENSIS

erc

European Research Council

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PRO-DEMOCRATIC

The model

1. A set of *candidates* or *projects* $C = \{c_1, c_2, \ldots, c_m\}$.
2. A set of voters $N = \{1, 2, \ldots, n\}$.

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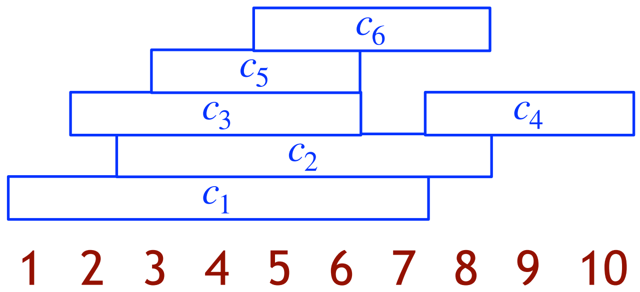

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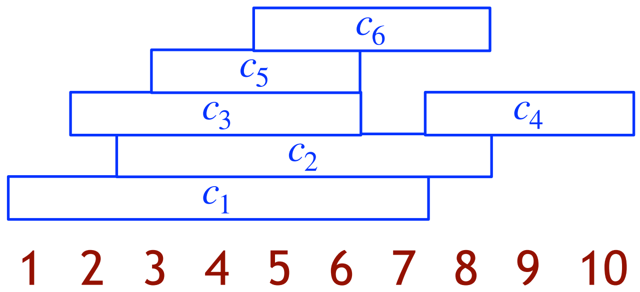

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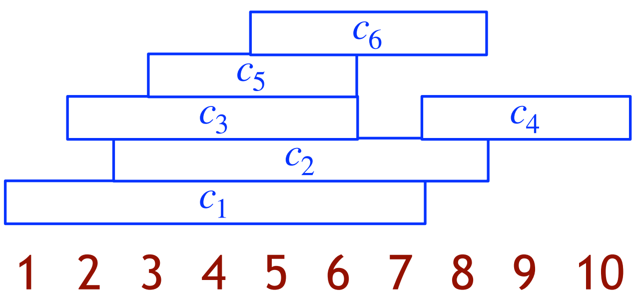
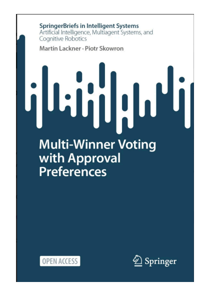


A subset of a given size k

committee elections



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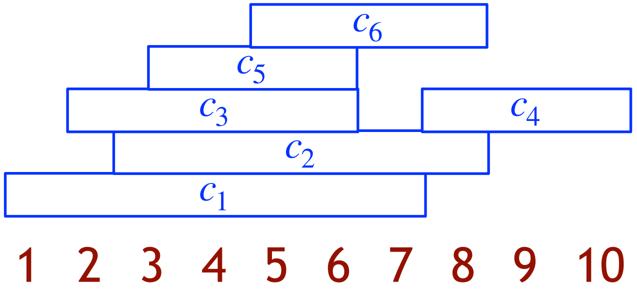

A subset of candidates with the total cost not exceeding the budget value b.

↓

participatory budgeting

S. Rey, J. Maly: The (Computational) Social Choice Take on Indivisible Participatory Budgeting, 2023.

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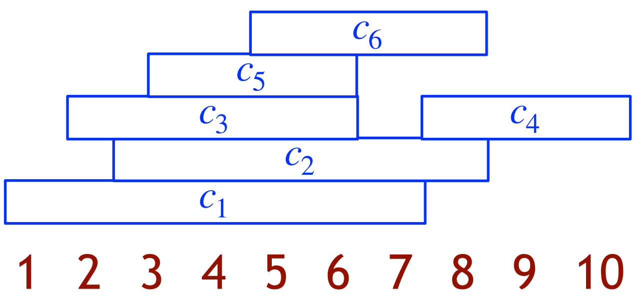

The candidates are grouped into pairs and foreach pair we need to select one.

↓

public decisions

V. Conitzer, R. Freeman, and N. Shah. Fair public decision making. EC-2017.

R. Freeman, A. Kahng, and D. M. Pennock. Proportionality in approval-based elections with a variable number of winners. IJCAI-2020.

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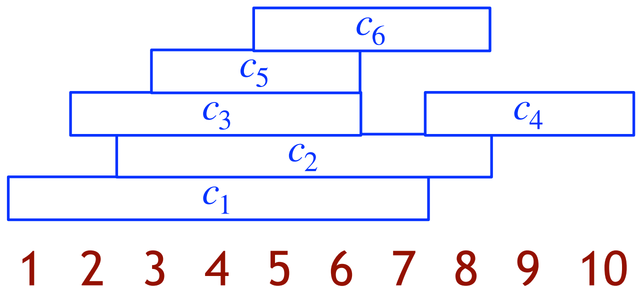

A subset of a given size k with **diversity constraints.**

L. E. Celis, L. Huang, and N. K. Vishnoi. Multiwinner voting with fairness constraints. IJCAI-2018.

R. Bredereck, P. Faliszewski, A. Igarashi, M. Lackner, and P. Skowron. Multiwinner elections with diversity constraints. AAAI-2018.

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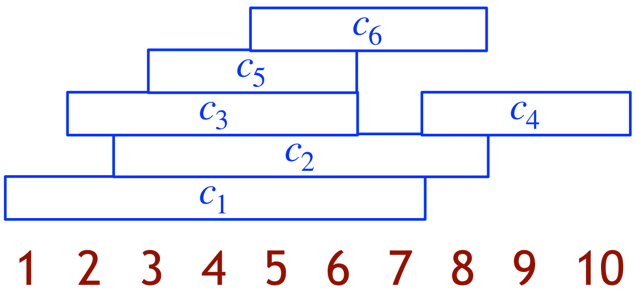

Ranking candidates

For each pair, c_1 and c_2, we introduce an auxiliary candidate $c_{1,2}$, whose selecting corresponds to ranking c_1 before c_2.

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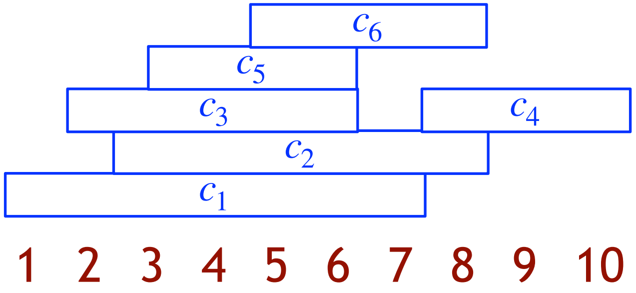

Committee elections with negative votes

For each c we introduce an auxiliary candidate $\bar{c}$, whose selecting corresponds to not selecting c.

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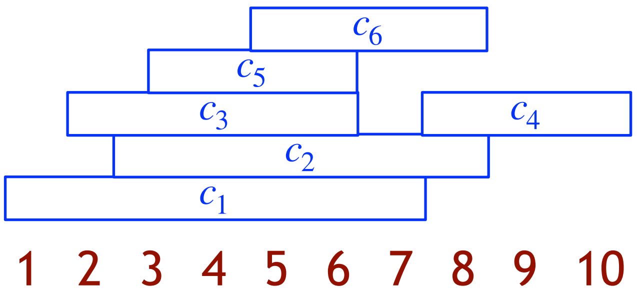

Our general model

We are given a nonempty **family of feasible sets** $\mathscr{F} \subseteq 2^C$.

($\mathscr{F}$ is closed under inclusions)

Proportionality in the general model

For committee elections:

An ℓ-cohesive group: a group of voters $S \subseteq N$ is cohesive if

(1) $|S| \geq \ell \cdot n/k$, and (2) $|\bigcap_{i \in S} A_i| \geq \ell$.

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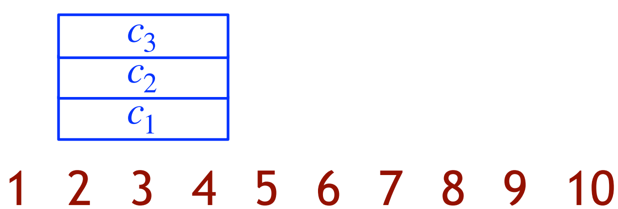
$k = 10$



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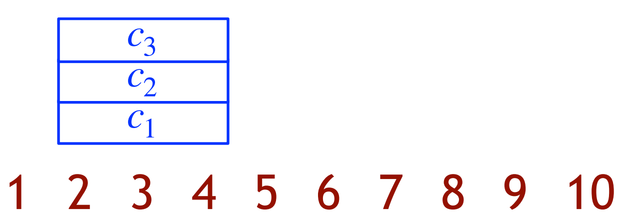

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The challange is how to properly define

ℓ-cohesiveness in the general model.

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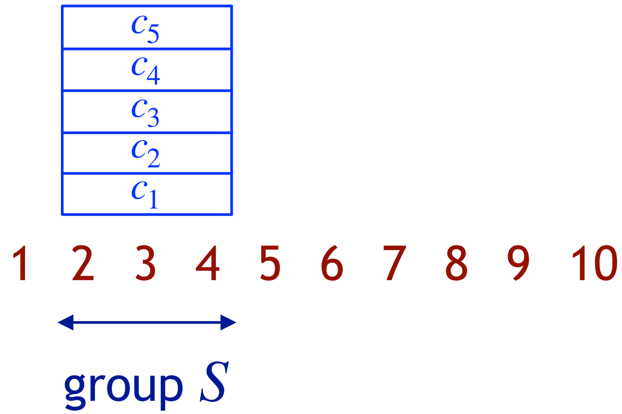

Group S agrees on some ℓ candidates.

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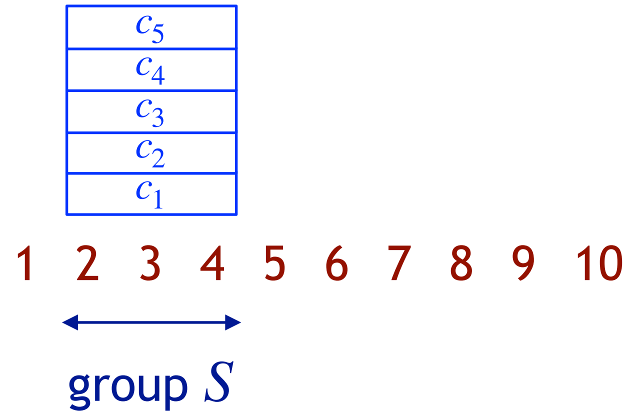

Group S agrees on some ℓ candidates. Do they deserve ℓ candidates?

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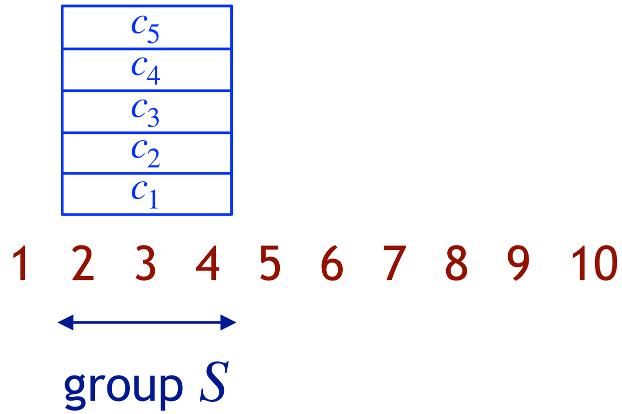

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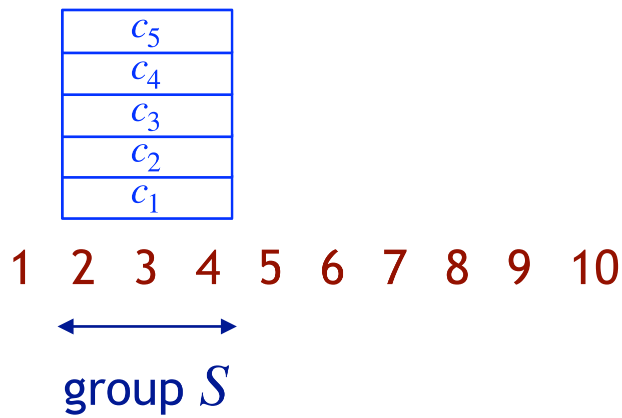

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$$\frac{\ell}{|S|} > \frac{|T|}{n - |S|}.$$

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To get EJR in the definition of ℓ-cohesiveness we look only at $T \subseteq W$.

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2. If $|T| > 7$ then

$$\frac{|S|}{n} = 0.3 > \frac{3}{3 + |T|}$$

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Example (committee elections with 50% of men and 50% of women):

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The group S is entitled to 30% of 50 that is to 15 candidates.
(The hardest T consists of 35 woman.)

Proportionality in the general model

Related work:

I.-A. Mavrov, K. Munagala, and Y. Shen. Fair multiwinner elections with allocation constraints. EC-2023

This paper introduces Restrained EJR. However,

1. In this example it provides no guarantees to the group S.
2. Is implied by our definition of EJR.
3. In general might contradict Pareto-Optimality

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This definition of Base EJR (and so EJR) implies:

1. EJR in the model of committee elections.

H. Aziz, M. Brill, V. Conitzer, E. Elkind, R. Freeman, and T. Walsh. Justified representation in approval-based committee voting. Social Choice and Welfare. 2017.

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2. **Strong EJR** in the model of sequential decision making.

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3. **Proportionality for cohesive groups** in the model of public decisions.

    P. Skowron and A. Górecki. Proportional public decisions. AAAI-2022.

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We can use this idea to extend other notions of propotionality.

(Base) EJR

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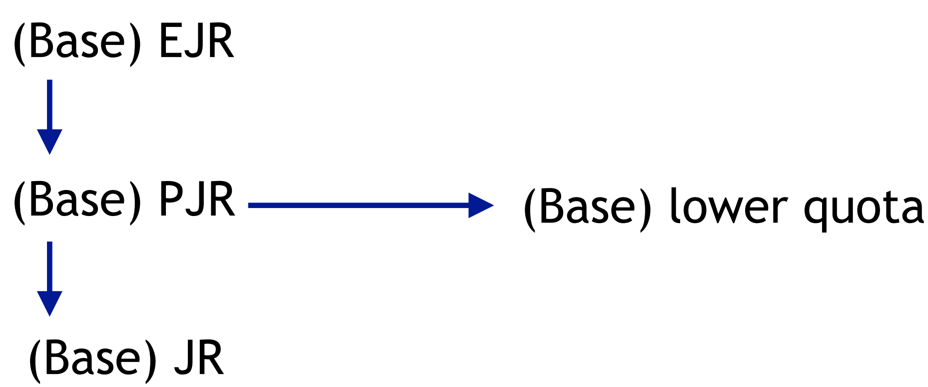

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(Base) EJR → (Base) PJR → (Base) JR

(Base) EJR → (for matroids) → (Base) proportionality degree of $\frac{\ell - 1}{2}$

(Base) PJR → (Base) lower quota

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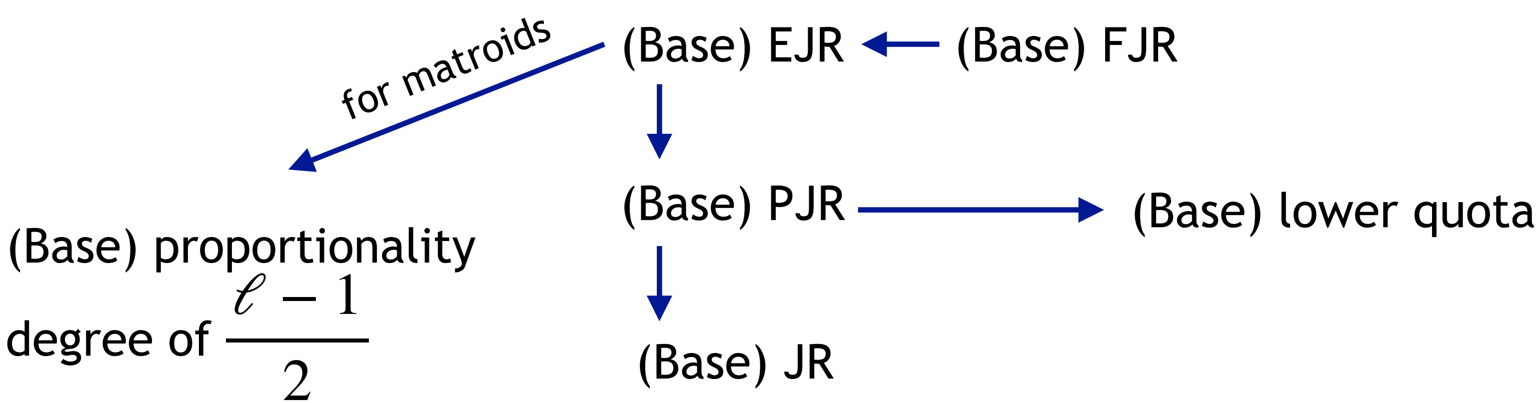

Proportionality in the general model

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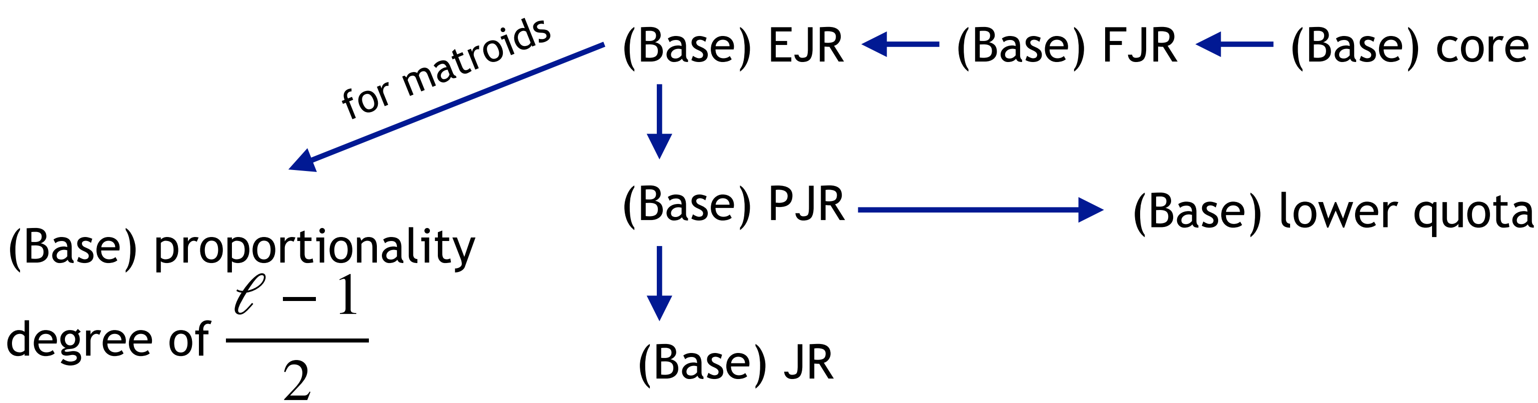

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Proportional Approval Voting (PAV): select an outcome W that maximizes :

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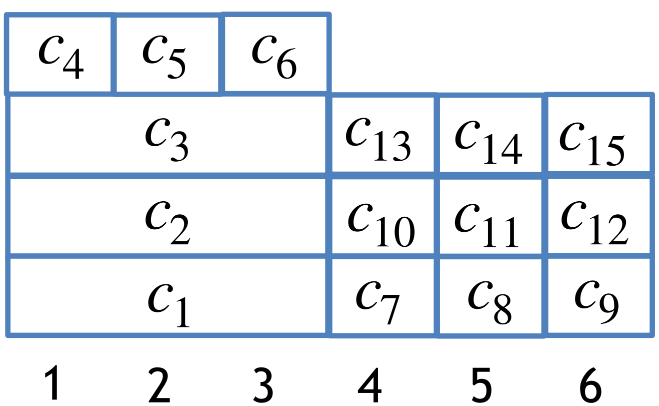

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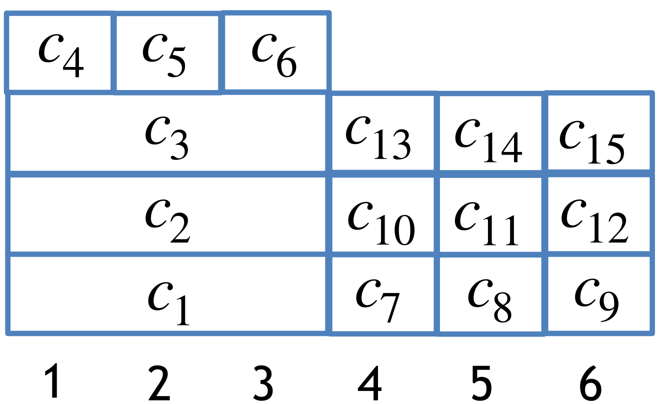

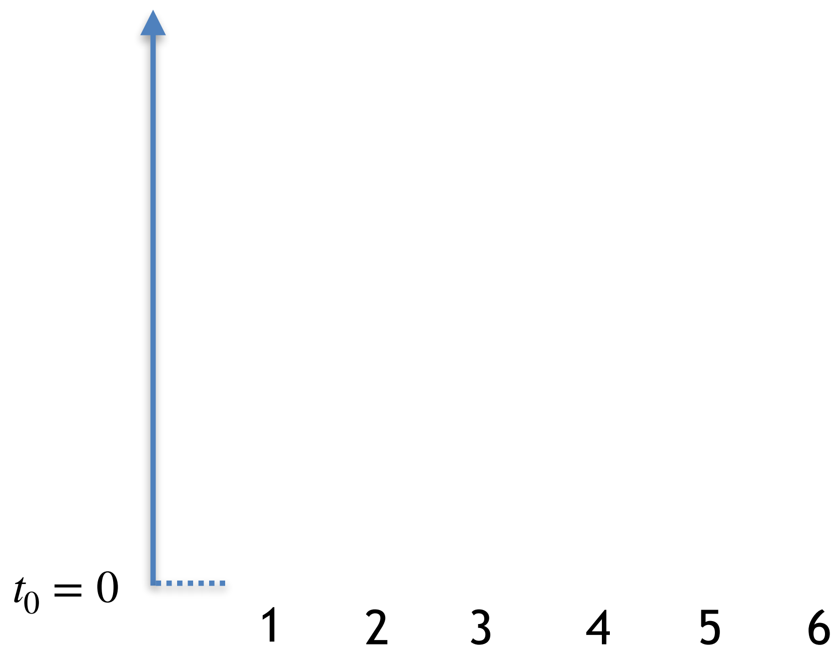

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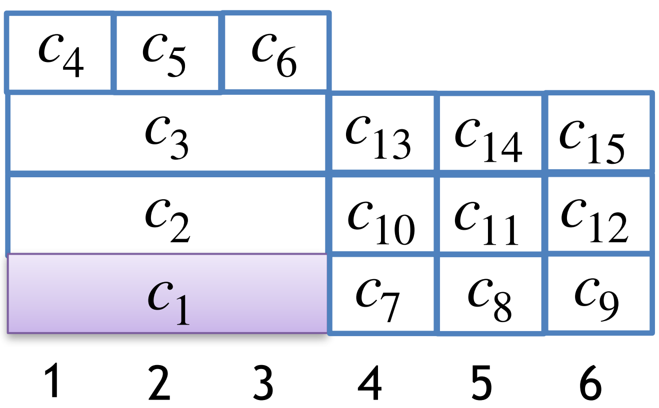

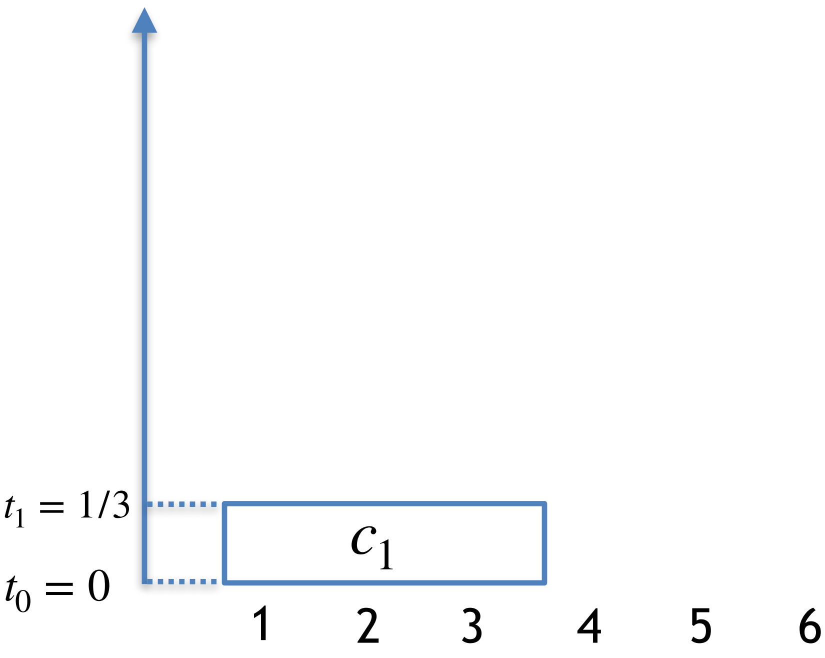

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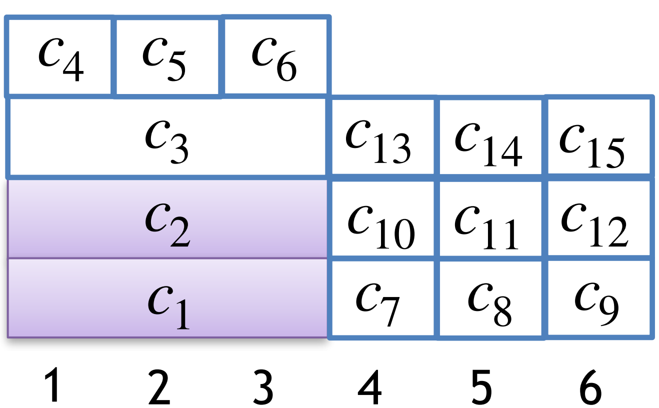

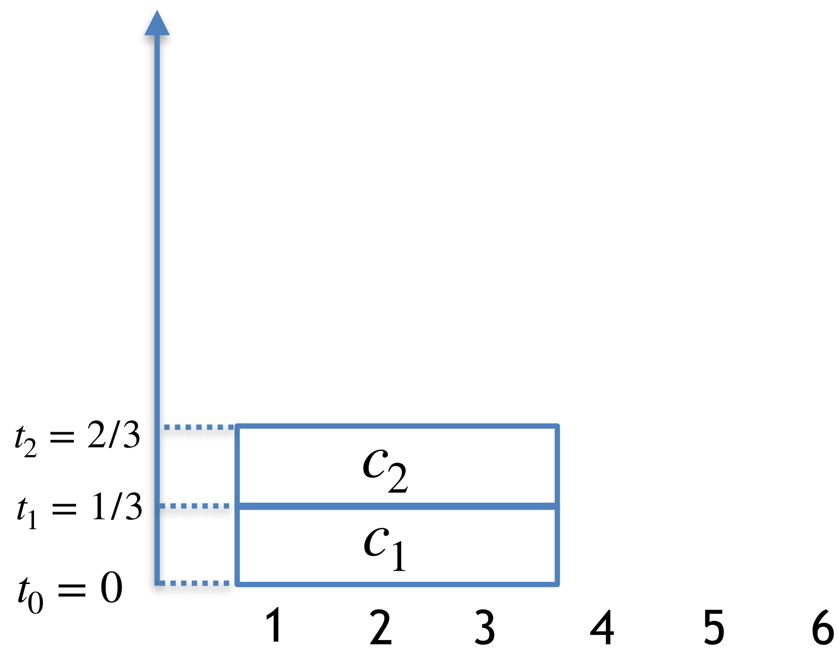

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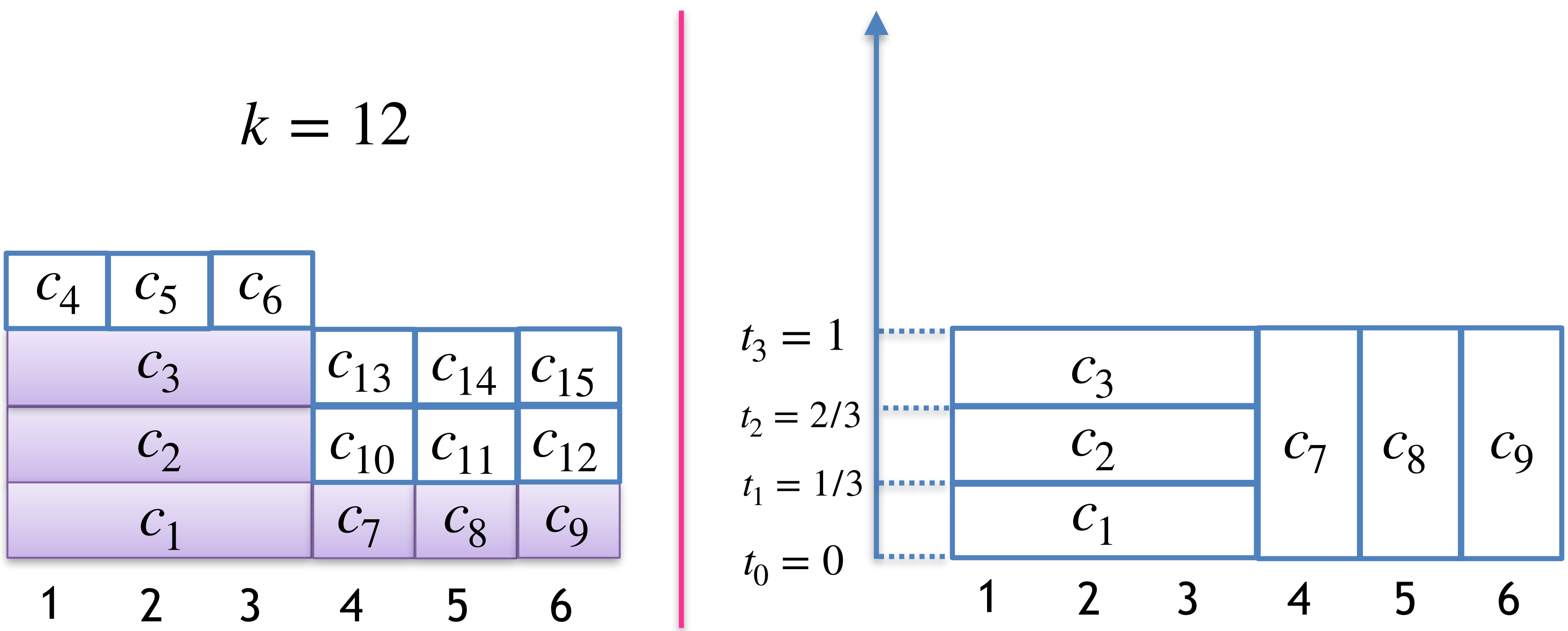

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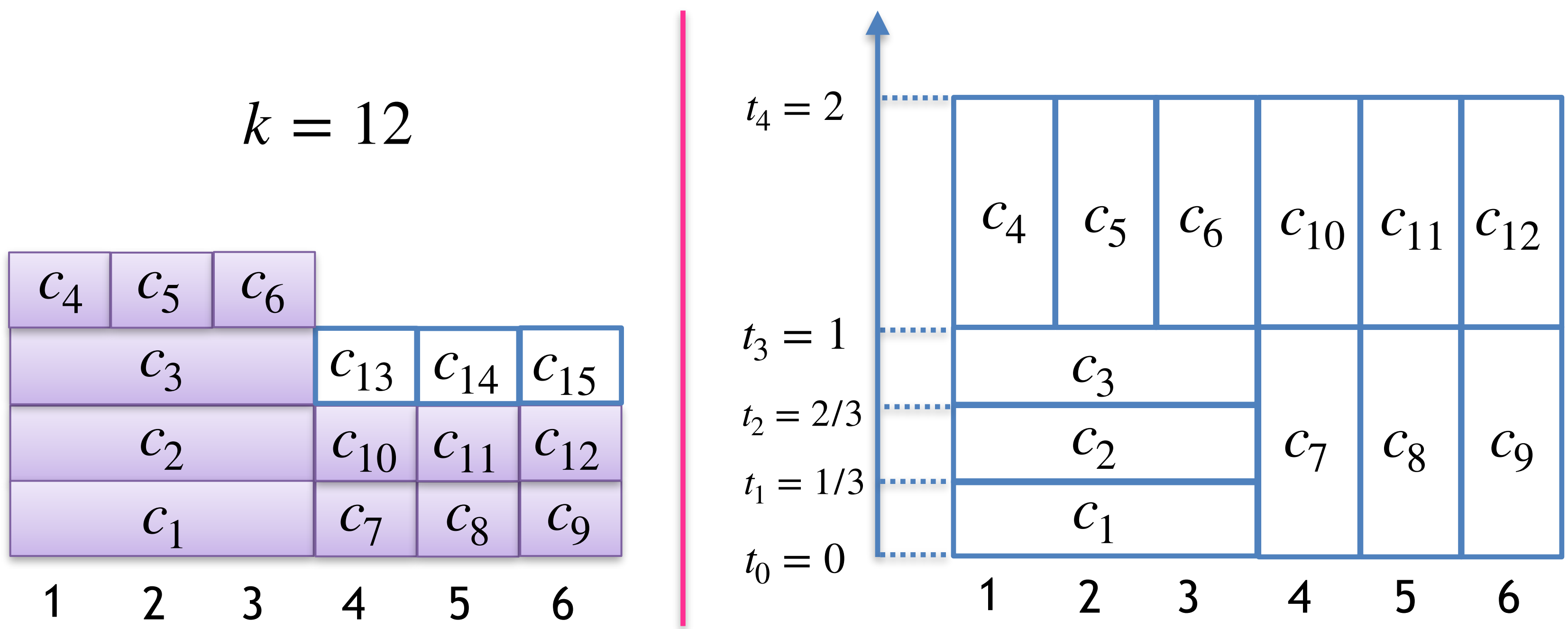

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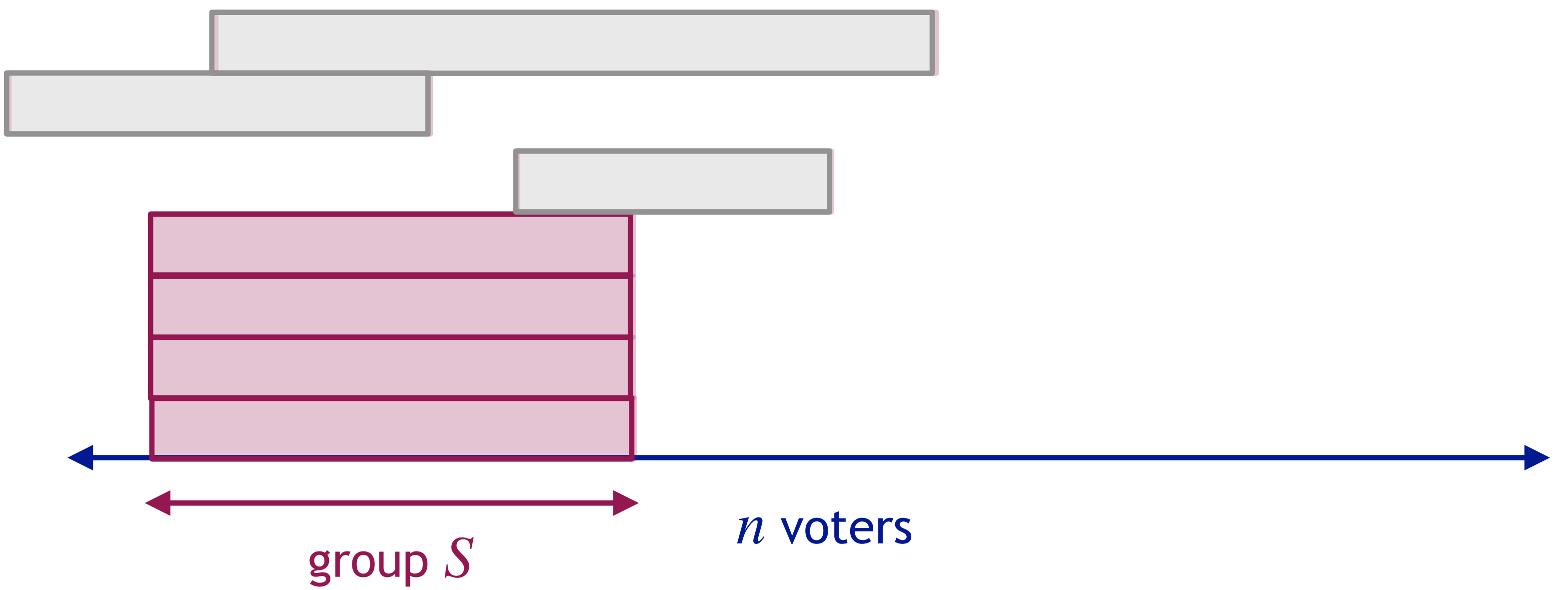

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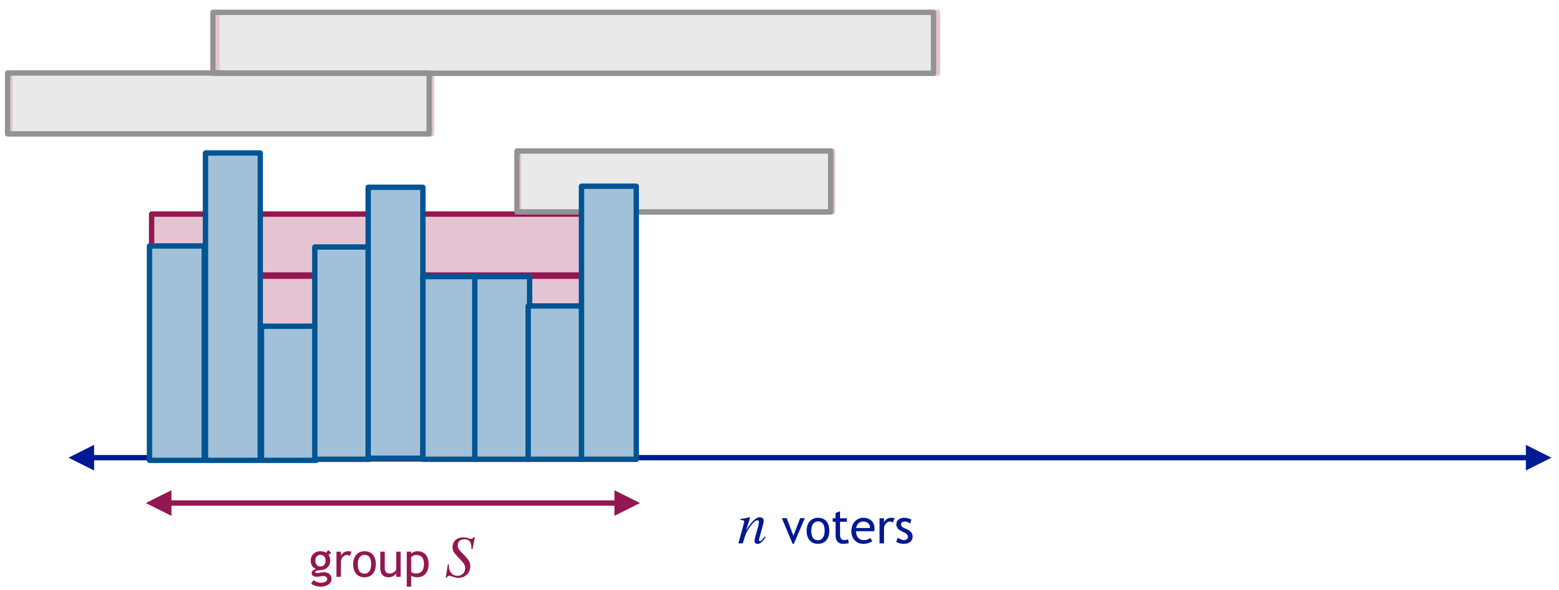

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6. **Theorem**: Stable priceability implies EJR if $\mathcal{F}$ is a matroid.

D. Peters, G. Pierczyński, N. Shah, and P. Skowron. Market-based explanations of collective decisions. I AAAI-2021.

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