Proportionality-Based Fairness in Social Choice

Part 2: Participatory Budgeting and other applications

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Recap of Part 1

- Apportionment
- Multi-winner voting: elect $k$ out of $m$ candidates
  - Proportionality axioms
    - Guarantees for cohesive groups: PJR, EJR, proportionality degree
    - Equal voting power: priceability
    - Guarantees for all groups: core
- Voting methods
  - Proportional Approval Voting (PAV)
  - Sequential Phragmén
  - (this part:) Method of Equal Shares
Overview of Part 2

- Application: Participatory Budgeting
  - Method of Equal Shares
- Multi-winner voting with rankings as input
- Sequential Decisions
- Fair mixing (a continuous/divisible model)
What is Participatory Budgeting (PB)?

Origins

- **Most generally:** Letting citizens have a say how government spends its money.
- Emerged in 1990s in Brasil, then spread through South America. Most commonly via discussion and deliberation in neighborhood plenary meetings.
- Used on level of cities, schools, housing complexes.

_Yves Cabannes_. “Participatory budgeting: a significant contribution to participatory democracy”. In: _Environment and Urbanization_ 16.1 (2004), pp. 27–46
What is Participatory Budgeting (PB)?

Current PB in Europe and North America

1. City government/parliament designates fixed budget for PB
2. Residents are invited to submit project proposals
3. City officials decide if projects are in scope, suggest changes, merge similar proposal, shortlist projects
4. Residents vote over projects (approval, online and/or paper)
5. Greedily take projects with highest score until budget runs out
6. City officials oversee implementation

Some PB Implementations

<table>
<thead>
<tr>
<th>City</th>
<th>Years</th>
<th>Budget</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Madrid</td>
<td>2018–19</td>
<td>EUR 100m</td>
<td>Knapsack votes, city+district</td>
</tr>
<tr>
<td>Madrid</td>
<td>2022</td>
<td>EUR 50m</td>
<td>Knapsack votes, city+district</td>
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<tr>
<td>Barcelona</td>
<td>2021</td>
<td>EUR 30m</td>
<td>Knapsack votes, 2 districts</td>
</tr>
<tr>
<td>Paris</td>
<td>2016–19</td>
<td>EUR 100m</td>
<td>4-approval, city+district</td>
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<tr>
<td>Paris</td>
<td>2021–22</td>
<td>EUR 75m</td>
<td>Majority judgment, unit cost</td>
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<tr>
<td>Lyon</td>
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<td>EUR 12.5m</td>
<td>10-approval</td>
</tr>
<tr>
<td>Strasbourg</td>
<td>2021</td>
<td>EUR 2m</td>
<td>Distribute 5 points to projects</td>
</tr>
<tr>
<td>Cambridge MA</td>
<td>2015–22</td>
<td>USD 1m</td>
<td>5-approval</td>
</tr>
<tr>
<td>New York City</td>
<td>2015–22</td>
<td>USD 40m</td>
<td>5-approval</td>
</tr>
<tr>
<td>Montreal</td>
<td>2021</td>
<td>CAD 25m</td>
<td>5-approval</td>
</tr>
<tr>
<td>Reykjavík</td>
<td>2021</td>
<td>IKR 850m</td>
<td>Knapsack vote+ “star” 1 project</td>
</tr>
<tr>
<td>Warsaw</td>
<td>2016–19</td>
<td>PLN 65m</td>
<td>Knapsack votes, city+district</td>
</tr>
<tr>
<td>Warsaw</td>
<td>2020–22</td>
<td>PLN 85m</td>
<td>10-approval, 15-approval district</td>
</tr>
<tr>
<td>Częstochowa</td>
<td>2022</td>
<td>PLN 10m</td>
<td>Distribute 10 points to projects</td>
</tr>
<tr>
<td>Gdansk</td>
<td>2022</td>
<td>PLN 20m</td>
<td>Distribute 5 points to projects</td>
</tr>
<tr>
<td>Krakow</td>
<td>2022</td>
<td>PLN 28m</td>
<td>Rank 3 projects, Borda scores</td>
</tr>
<tr>
<td>Portugal</td>
<td>2018</td>
<td>EUR 5m</td>
<td>1 vote nation-wide, 1 vote region</td>
</tr>
</tbody>
</table>

Next slide: formal model
Basic Model

Let $N = \{1, \ldots, n\}$ be the set of voters.

Let $C = \{c_1, \ldots, c_m\}$ be the set of projects.

Each project $c_j$ has a cost: $\text{cost}(c_j) \geq 0$.

For $T \subseteq C$, write $\text{cost}(T) = \sum_{c \in T} \text{cost}(c)$.

Budget limit $B \geq 0$.

An outcome is a set $W \subseteq C$ that is affordable: $\text{cost}(W) \leq B$.

Each voter submits an approval ballot, a set $A_i \subseteq C$ of projects that $i \in N$ approves.

Could be enriched with additional feasibility constraints, utilities (additive, non-additive, negative, etc.) . . .

If $\text{cost}(c) = 1$ for all $c \in C$, and $B \in \mathbb{N}$, we are in the unit cost case $\rightarrow$ committee elections.
Approval Ballots

In almost all implementations, approval ballots are used.

— Ballot Paper —

Total available budget: €3,000,000.
Approve up to 4 projects.

- Extension of the Public Library
  Cost: €200,000

- Photovoltaic Panels on City Buildings
  Cost: €150,000

- Bicycle Racks on Main Street
  Cost: €20,000

- Sports Equipment in the Park
  Cost: €15,000

- Renovate Fountain in Market Square
  Cost: €65,000

- Additional Public Toilets
  Cost: €340,000

- Digital White Boards in Classrooms
  Cost: €250,000

- Improve Accessibility of Town Hall
  Cost: €600,000

- Beautiful Night Lighting of Town Hall
  Cost: €40,000

- Resurface Broad Street
  Cost: €205,000

✗
✗
✗
Circleville

\[ B = $400k \]

<table>
<thead>
<tr>
<th></th>
<th>Cost</th>
<th>Votes</th>
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<tbody>
<tr>
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<td>$150k</td>
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<tr>
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<tr>
<td>C</td>
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<td>120k</td>
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<tr>
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<td>F</td>
<td>$10k</td>
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<tr>
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<td>$90k</td>
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<tr>
<td>H</td>
<td>$60k</td>
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<tr>
<td>I</td>
<td>$4k</td>
<td>110k</td>
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<tr>
<td>J</td>
<td>$70k</td>
<td>80k</td>
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<td>K</td>
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<tr>
<td>R</td>
<td>$10k</td>
<td>90k</td>
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<tr>
<td>S</td>
<td>$40k</td>
<td>90k</td>
</tr>
<tr>
<td>T</td>
<td>$7k</td>
<td>90k</td>
</tr>
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</table>
There is a simple, polynomial time method that enjoys strong proportionality properties.

- **Equally** split the budget between voters.
- Look for project whose approvers can pay for it.
- Buy project, and share the cost **equally** between its approvers.
- If there are several options, take the one where we can spread the cost most thinly.
Method of Equal Shares: Example

Budget $B = \$100$, 10 voters, everyone starts with $\$10$.

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<thead>
<tr>
<th></th>
<th>cost</th>
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<th>$i_2$</th>
<th>$i_3$</th>
<th>$i_4$</th>
<th>$i_5$</th>
<th>$i_6$</th>
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Equal Shares will select Project 3, then Project 5, then terminate.
Method of Equal Shares: More Popular is Better

Budget $B = $100, 10 voters, everyone starts with $10.

**Project 1 with cost $35**

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<tr>
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</table>

$\rho = 7$

**Project 2 with cost $35$ but more approvers**

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<tr>
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<th>$i_3$</th>
<th>$i_4$</th>
<th>$i_5$</th>
<th>$i_6$</th>
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<td>5.8</td>
<td>5.8</td>
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<td>10</td>
<td>10</td>
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</table>

$\rho = 5.8$
Method of Equal Shares: Cheaper is Better

Budget $B = $100, 10 voters, everyone starts with $10.

**Project 1 with cost $35**

<table>
<thead>
<tr>
<th>10</th>
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<tr>
<td>$i_1$</td>
<td>$i_2$</td>
<td>$i_3$</td>
<td>$i_4$</td>
<td>$i_5$</td>
<td>$i_6$</td>
<td>$i_7$</td>
<td>$i_8$</td>
<td>$i_9$</td>
<td>$i_{10}$</td>
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</table>

$\rho = 7$

**Project 3 with cost $25 with same approvers**

<table>
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<tr>
<th>10</th>
<th>10</th>
<th>10</th>
<th>10</th>
<th>10</th>
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<tbody>
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<td>5</td>
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</tr>
<tr>
<td>$i_1$</td>
<td>$i_2$</td>
<td>$i_3$</td>
<td>$i_4$</td>
<td>$i_5$</td>
<td>$i_6$</td>
<td>$i_7$</td>
<td>$i_8$</td>
<td>$i_9$</td>
<td>$i_{10}$</td>
</tr>
</tbody>
</table>

$\rho = 5$
Implement Project 3, so $i_1, \ldots, i_5$ each spend $5$. 

\[
\begin{array}{cccccccc}
 i_1 & i_2 & i_3 & i_4 & i_5 & i_6 & i_7 & i_8 & i_9 & i_{10} \\
 5 & 5 & 5 & 5 & 5 & 10 & 10 & 10 & 10 & 10 \\
\end{array}
\]
Method of Equal Shares: Richer is Better

Project 4 with cost $24

\[ \rho = 7 \]

Project 5 with cost $24 but with richer approvers

\[ \rho = 6 \]
Method of Equal Shares: Completions

In the example, $B = $100, but Equal Shares only spends $25 + $24 = $49! Unselected projects 1, 2, and 4 each would still fit into the budget limit. (Equal Shares is not “exhaustive”.)

Intuition: Equal Shares only spends money if fairness forces it to.

Completion strategies:

- Completion by varying the budget: repeatedly increase the budget by $1 until the next increment would make the output infeasible.
- Completion by utilitarian: continue using standard greedy.
- Other proposals: completion by Phragmén’s method, completion by perturbation.
- Do not complete, save money (maybe for next year).
Method of Equal Shares: Varying the budget

Pabulib data, Warsaw 2021 election.

x-axis: input budget. y-axis: cost of Equal Shares outcome
Equal Shares satisfies EJR

Theorem

For approval utilities, the outcome of Equal Shares satisfies Extended Justified Representation:

If $S \subseteq N$ is a group of voters, and $T \subseteq C$ a proposal that

- $S$ can afford: $\text{cost}(T) \leq B \cdot |S|/|N|$, and
- $S$ unanimously approves: $T \subseteq \bigcap_{i \in S} A_i$,

then at least 1 voter in $S$ approves at least $|T|$ projects in the Equal Shares outcome, so $u_i(W) \geq u_i(T)$.

- The only natural method known to satisfy EJR.
- **Proof idea:** The projects in set $T$ offer good “bang per buck” to $S$. While voters have not reached their budget limit, Equal Shares always selects the projects with the best bang per buck. Consider the first agent in $S$ whose money runs out: that agent spent her money so effectively that she gets enough utility.
Overview of multi-winner methods

The Method of Equal Shares can also be used for committee elections \((B = k)\), and thus we can compare it to PAV and to Sequential Phragmén.

<table>
<thead>
<tr>
<th></th>
<th>PAV</th>
<th>Sequential Phragmén</th>
<th>Equal Shares</th>
</tr>
</thead>
<tbody>
<tr>
<td>priceable</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>PJR</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>EJR</td>
<td>✓</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>core approximation</td>
<td>2-approx.</td>
<td>?</td>
<td>(O(\log k))-approx.</td>
</tr>
<tr>
<td>welfarist</td>
<td>✓</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Pareto-optimal</td>
<td>✓</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>Pigou–Dalton</td>
<td>✓</td>
<td></td>
<td>✓</td>
</tr>
<tr>
<td>computation</td>
<td>NP-complete</td>
<td>polynomial time</td>
<td>polynomial time</td>
</tr>
<tr>
<td>extends to PB</td>
<td>✗</td>
<td>✓</td>
<td>✓ ✓</td>
</tr>
</tbody>
</table>
Method of Equal Shares for additive utilities

Equal Shares can also be defined for general additive valuations, with $u_i(c) \geq 0$ being the utility $i$ assigns to project $c$.

Idea: voters contribute to the cost of a project in proportion to their utility, so voter $i$ pays $\rho \cdot u_i(c)$ for $c$, for some $\rho$ such that

$$\sum_{i \in N} \min\{\rho \cdot u_i(c), \text{ i’s remaining budget}\} = \text{cost}(c).$$

If a project has smaller $\rho$ then it offers better bang-per-buck.

Equal Shares at each step selects a project with minimum $\rho$.

**Theorem**

For additive utilities, Equal Shares satisfies EJR up to 1 project (EJR1).

- For general additive utilities, satisfying EJR is weakly NP-hard.
- But there is an existence proof for EJR.

Next slide: cost utilities
Method of Equal Shares for cost utilities

Standard approval-based (0/1) Equal Shares selects projects in order of (# of votes)/cost. But most cities want to go by just # of votes.

Solution: use cost utilities with Equal Shares:

\[ u_i(c) = \begin{cases} 
  \text{cost}(c) & \text{if } c \in A_i, \\
  0 & \text{if } c \notin A_i.
\end{cases} \]

Simple explanation of Equal Shares with cost utilities:

- Repeatedly select project with highest number of votes, provided its supporters have enough money for it.
- Split the cost equally among the supporters.
- If a voter runs out of money, delete the voter and update the vote counts.*

* if a voter has very little money left, the voter will only count fractionally when calculating vote counts.
Data: Utilitarian Welfare
Warsaw data, 2022, cost utilities

Actual outcome

Equal Shares

Difference
Data: What % of voters have positive utility?
Warsaw data, 2022, cost utilities

<table>
<thead>
<tr>
<th>Actual outcome</th>
<th>Equal Shares</th>
<th>Difference</th>
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<tbody>
<tr>
<td>85%</td>
<td>91%</td>
<td>6%</td>
</tr>
<tr>
<td>80%</td>
<td>80%</td>
<td>0%</td>
</tr>
<tr>
<td>88%</td>
<td>92%</td>
<td>4%</td>
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<td>83%</td>
<td>91%</td>
<td>2%</td>
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<td>90%</td>
<td>7%</td>
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<tr>
<td>91%</td>
<td>94%</td>
<td>1%</td>
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</tbody>
</table>

[Maps showing percentage distribution]
Data: What % of voters prefer Equal Shares outcome over actual outcome?
Warsaw data, 2022, number utilities
Wieliczka's “Green Million” (April 2023)

Method of Equal Shares

Standard voting method
Wieliczka’s “Green Million” (April 2023)

Method of Equal Shares

Standard voting method
Strategyproofness


- Proportional rules must be manipulable.
- Voters can pretend to not approve popular projects.
- With unit costs, the greedy method is strategyproof, but not true in PB.
The **Method of Equal Shares** is a fairer voting rule for participatory budgeting. It provides proportional representation and allows every voter to decide about an equal part of the budget.

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**Method of Equal Shares in Cities in 2023**

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**NEW**

**Motivation**

The method is an alternative to the knapsack algorithm which is used by most cities even though it is a disproportional method. For example, if 51% of the population support 10 red projects and 49% support 10 blue projects, and the money suffices only for 10 projects, the knapsack budgeting will choose the 10 red supported by the 51%, and ignore the 49% altogether. In contrast, the method of equal shares would pick 5 blue and 5 red projects.

The method guarantees proportional representation: it satisfies the strongest known variant of the justified representation axiom that is known to be satisfiable in participatory budgeting.

**Intuitive explanation**

In the context of participatory budgeting the method assumes that the municipal budget is initially evenly distributed among the voters. Each time a project is selected its cost is divided among those voters who supported the project and who still have money. The savings of these voters are decreased accordingly. If the voters vote via approval ballots, then the cost of a selected project is distributed equally among the voters; if they vote via cardinal ballots, then the cost is distributed proportionally to the utilities the voters enjoy from the project. The rule...
Extensions I

- **Understand voting data**: how to identify groups with similar interests?
- **Negative votes**: Allow voters to vote against a project. Some might not want a particular project to be implemented near them. Also useful for non-PB applications of the same model, e.g. allowing downvotes.
- **“At most one of these” constraints**: Empty plot of land, many projects that could be implemented there. Equal Shares has a natural generalization but does not satisfy EJR anymore.


- **Arbitrary constraints**: Allow arbitrary constraints on the collection of feasible sets. Perhaps go via judgment aggregation (JA). [And import proportionality to JA!]

Extensions II

- **Time**: PB is often repeated every year. We might want to be fair to people/groups over the long term, or we could allow people to invest or to save money for future years.


- **Allow for some divisible projects. Milestones.**

- **Multiknapsack.** Allow for several budget constraints simultaneously, such as money and time and CO2e emissions. (Or money and number of winning projects.)

- **Separate budgets.** In practice, voters vote for several budgets at the same time (city-wide, district) with disjoint project proposals. Is there something better than running the same voting rule separately for each?

- **Agent contributions**: Allow agents to add their own money to the budget.


Multi-Winner Election with Ranking Input

▶ **Single Transferable Vote**: a proportional method used for Australia’s Senate, in Ireland, Malta, and in several local elections.

▶ Informal description: Set budget $B = k$ to the number of seats, split budget equally between voters, it costs 1 to elect a candidate. Repeatedly look for a candidate that can be purchased using the money of voters who rank it top. If no such candidate exists, delete the worst candidate and look again.

▶ This satisfies a proportionality axiom somewhat similar to PJR and friends, but quite weak, called **Proportionality for Solid Coalitions** (PSC).

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▶ Recent work: Define new rules that satisfy stronger proportionality properties (including “rank-PJR”) and better monotonicity properties – including a variant of the Method of Equal Shares.

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Consider sequential decision making, where in each of several rounds \( j \in R = \{1, \ldots, T\} \), there is a set of available alternatives \( C_j \) (which may differ from round to round), and a set \( N \) of voters (the same across rounds) who say in each round which candidates they approve. (But can also consider rankings or utilities.)

We want a fair voting rule that makes these decisions in each round. If we require the rule to be online, this is known as perpetual voting.

One can define analogues of Phragmén, Equal Shares, and PAV for this setting, and they satisfy analogues of PJR and EJR (“if a group of \( \alpha \)% of the voters agree in every round, then they approve the decision in \( \alpha \)% of the rounds”).

Other approaches to this problem focus on maximizing the Nash welfare across rounds.

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**Martin Lackner.** “Perpetual voting: Fairness in long-term decision making”. In: *Proceedings of the 34th AAAI Conference on Artificial Intelligence (AAAI)*. 2020, pp. 2103–2110

**Martin Lackner and Jan Maly.** “Proportional Decisions in Perpetual Voting”. In: *Proceedings of the 37th AAAI Conference on Artificial Intelligence (AAAI)*. 2023

**Nikhil Chandak, Shashwat Goel, and Dominik Peters.** “Proportional Aggregation of Preferences for Sequential Decision Making”. In: *2023*

**Rupert Freeman, Seyed Majid Zahedi, and Vincent Conitzer.** “Fair social choice in dynamic settings”. In: *Proceedings of the 26th International Joint Conference on Artificial Intelligence (IJCAI)*. 2017, pp. 4580–4587
Proportional Rankings

- House monotonic rules like Sequential Phragmén construct the committee one-by-one. We can view its output as a ranking that is proportional. In particular, each prefix is proportional.

- Many applications: aggregating search results, combining party lists, listing topics in online deliberation platforms, vote systems for Q&A sessions


Fair mixing

- **Continuous** problem: given preferences, decide on a mixture \( p = (p_c)_{c \in C} \) between alternatives with \( \sum_{c \in C} p_c = 1 \).

- Applications: divide a society’s budget between projects; conference committee deciding how much time to assign to talks / posters / breaks; sharing time between group activities.

- Good rule: pick the distribution maximizing Nash welfare \( \prod_{i \in N} u_i(p) \): this satisfies the core and an analogue of EJR. (In a way, this rule is similar to PAV.)

- For approvals, there is also a *strategyproof* rule that satisfies an analogue of PJR (but fails Pareto-optimality): The conditional utilitarian rule.

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Proportionality-Based Fairness in Social Choice
Part 2: Participatory Budgeting and other applications

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