

# Sparsity — homework 2

Measuring sparsity and introduction to generalized coloring numbers

Deadline: November 14th, 2019, 23:00 CET

**Problem 1.** Prove that if  $\mathcal{C}$  is a somewhere dense class of graphs that is closed under taking subgraphs, then the class  $\mathcal{C}'$  consisting of all bipartite graphs contained in  $\mathcal{C}$  is also somewhere dense.

**Problem 2.** A graph  $G$  is called *k-planar* if there is a drawing of  $G$  in the plane in which the edges may intersect, but the following conditions are satisfied:

- every pair of edges intersects in at most one point, which moreover is not an endpoint of any of them;
- no three edges intersect at one point; and
- every edge intersects at most  $k$  other edges.

Prove that for every  $k \in \mathbb{N}$ , the class of  $k$ -planar graphs has bounded expansion.

**Problem 3.** Prove that if  $\mathcal{C}$  is the class of all graphs of maximum degree at most 3, then there exists a constant  $\varepsilon > 0$  such that

$$\text{wcol}_d(\mathcal{C}) \geq 2^{\varepsilon d} \quad \text{for every } d \in \mathbb{N}.$$

*Note: At least 5 points will be awarded for solutions assuming that  $\mathcal{C}$  is the class of graphs of maximum degree at most  $\Delta$  for some constant  $\Delta \geq 4$ .*