Parameterized algorithms — tutorial 3

Dynamic programming on subsets, iterative compression

Problem 1. We are given universe U and family of its subsets $\mathcal{F}$. Prove that following problems can be solved in $O(2^{|U|}|\mathcal{F}|)$ time:

- Set Packing: Find the biggest subfamily of $\mathcal{F}$ of pairwise disjoint sets
- Partitioning: Determine whether U is a sum of disjoint sets from $\mathcal{F}$

Problem 2. In Subset TSP problem we are given a weighted directed graph G and set of terminals $T \subseteq V(G)$. We are asked what is the shortest closed walk passing through every terminal at least once. Prove that this problem can be solved in time complexity $\mathcal{O}^\star(2^{|T|})$.

Problem 3. In Steiner Tree problem we are given weighted undirected graph G and a set $T \subseteq V(G)$ of terminals and we are asked to find connected subgraph containing all terminals with the least possible sum of its edges weight. Prove that if in Steiner Tree problem we are additionally given that G is planar and we are given its embedding on a plane so that all terminals lie on an outer face then we can solve it in polynomial time.

Problem 4. We are given a group of n friends who have a long history of common expenses paid by various people or little loans between each other. We are asked what is the lowest number of money transfers that these friends need to perform so that there are no more debts between them. For example if Alice lent Bob 2 dollars, then Bob lent Carol 3 dollars and then Alice lent Bob 1 dollar more then answer is 1 since Carol can return 3 dollars to Alice. Prove that this problem can be solved in $\mathcal{O}^\star(3^n)$ time. (During our course we will learn how to improve it to $\mathcal{O}^\star(2^n)$ by using Fast Subset Convolution)

Problem 5. In d-Hitting Set problem we are given a universe U, family of its subsets $\mathcal{F}$ so that $\forall_{A\in\mathcal{F}}|A| \leqslant d$ and an integer k and we are asked whether there exists $X \subseteq U$ of size at most k such that $X \cap A \neq \emptyset$ for every $A \in \mathcal{F}$.

Prove that for every constant c, if we can solve Vertex Cover problem in time $\mathcal{O}^\star(c^k)$ then we can solve 3-Hitting Set in time $\mathcal{O}^\star((1+c)^k)$

Problem 6. By definiton, iterative compression adds factor n to running time complexity. Show that this factor can be reduced to k for Vertex Cover and Feedback Vertex Set in Tournaments problems.

Problem 7. In Cluster Vertex Deletion problem we are given a graph G, positive integer k and we are asked whether it is possible to remove at most k vertices in order to get a graph where every connected component is a clique. Prove that this problem can be solved in $\mathcal{O}^\star(2^k)$ time.

Problem 8. In Odd Cycle Transversal problem we are given a graph G and an integer k and we are asked whether there exists a set $X \subseteq V(G)$ of size at most k so that $G \setminus X$ is bipartite. Show that this problem can be solved in $\mathcal{O}^\star(3^k)$ time.