Parameterized algorithms — tutorial 10

LP guided branching

Problem 1. Prove that Vertex Cover Above Matching is FPT. In other words prove that if M is maximum matching in graph G then we can determine whether G admits a vertex cover of size k in time $f(k - |M|)|G|^{O(1)}$.

Problem 2. In following exercise we are going to improve algorithm for Vertex Cover Above LP.

Let us remind that as long as *all*-$\frac{1}{2}$ solution is not a unique solution to LPVC then we are able to find a different solution to LPVC that partitions $V = V_0 \cup V_{\frac{1}{2}} \cup V$ where $V_0, V_{\frac{1}{2}}$ and V_1 are disjoint and $V_{\frac{1}{2}} \neq V$, takes greedily whole V_1 to vertex cover, disregards V_0 and decreases our budget by $|V_1|$, or in other words reduces instance (G, k) to instance $(G[V_{\frac{1}{2}}], k - |V_1|)$.

For an independent set I let's define its *surplus* as $|N(I)| - |I|$.

1. Prove that after exhaustively applying mentioned reduction rule there is no independent set with nonpositive surplus.
2. Prove that we can either find in polynomial time an independent set of surplus 1 or determine that it doesn't exist.
3. Design a reduction rule that exploits existence of independent set of surplus 1. How does measure $\mu(G, k) = k - vc^*(G)$ change after this reduction?
4. Prove that if our graph doesn't contain independent set of surplus at most 1 then we can improve our estimates for $\mu(G, k)$ in our branching steps. Compute final complexity.

Problem 3. In Odd Cycle Transversal problem we are given a graph G and an integer k and we are asked whether there exists $X \subseteq V(G)$ so that $|X| \leqslant k$ and $G \setminus X$ is bipartite.

Let G be a graph and let H be a graph created in a following way. Let $V_i = \{v_i | v \in V(G)\}$ for $i \in \{1, 2\}$ and let vertex set of H be $V(H) = V_1 \cup V_2$. Let

$$E(H) = \{v_1 v_2 \mid v \in V(G)\} \cup \{u_i v_i \mid uv \in E(G), i \in \{1, 2\}\}.$$

Prove that G admits odd cycle transversal of size k iff H admits vertex cover of size $n + k$. Deduce that we can solve odd cycle transversal in time $O(2.6181^k n^{O(1)})$.

Problem 4. In Variable Deletion Almost 2-SAT problem we are given 2-CNF formula ϕ and an integer k and we are asked whether we can delete at most k variables from ϕ together with all clauses containing them in order to make ϕ satisfiable.

Prove that Variable Deletion Almost 2-SAT can be solved in $O(2.6181^k n^{O(1)})$ time.