Parameterized algorithms — tutorial 1

Branching algorithms

Problem 1. Prove that in any graph G there are at most 2^k inclusion-wise minimal vertex covers of size at most k, and they can be enumerated in time $2^k \cdot n^{\mathcal{O}(1)}$. Is the 2^k bound optimal?

Problem 2. Prove that Clique parameterized by $n - k$ is FPT.

Problem 3. Suppose $\mathcal{F}$ is a finite family of graphs. A graph G is $\mathcal{F}$*-free* if G does not contain any graph from $\mathcal{F}$ as an induced subgraph. In the $\mathcal{F}$-free Vertex Deletion problem we are given a graph G and integer k, and we ask whether one can remove at most k vertices from G to obtain an $\mathcal{F}$-free graph. Prove that there is a constant c, depending only on $\mathcal{F}$, such that $\mathcal{F}$-free Vertex Deletion can be solved in time $c^k \cdot n^{\mathcal{O}(1)}$. Prove the same for the $\mathcal{F}$-free Edge Deletion, $\mathcal{F}$-free Edge Completion, and $\mathcal{F}$-free Edge Editing problems, where instead of removing vertices we may remove edges, add edges, or add and remove edges.

Problem 4. Prove that for every constant $d \in \mathbb{N}$, the d-Regular Vertex Deletion problem, where given G and k we want to remove at most k vertices from G to obtain a d-regular graph, is FPT when parameterized by k.

Problem 5. A directed graph D is a *tournament* if between every pair of vertices there is exactly one arc. In Directed Feedback Vertex Set we are given a directed graph D and integer k, and we want to remove at most k vertices from D to obtain a DAG. Prove that DFVS on tournaments is FPT when parameterized by k.

Problem 6. Prove that for every constant $p \in \mathbb{N}$, the following problem can be solved in time $p^k \cdot \|\varphi\|^{\mathcal{O}(1)}$: given a boolean formula φ in p-CNF, decide whether there exists an assignment that satisfies φ and sets at most k variables to true.

Problem 7. Prove that the following problem can be solved in time $2^k \cdot \|\varphi\|^{\mathcal{O}(1)}$: given a boolean formula φ in CNF, decide whether there exists an assignment that satisfies at most k clauses of φ.

Problem 8. We consider the Independent Set problem: given G and k, decide whether there is a subset of k pairwise non-adjacent vertices in G. Prove that this problem can be solved on graphs of maximum degree 4 in time $2.31^k \cdot n^{\mathcal{O}(1)}$.

Problem 9. Prove that Triangle-free Vertex Deletion can be solved in time $2.562^k \cdot n^{\mathcal{O}(1)}$.

Problem 10. In the Closest String problem we are given strings $s_1, \ldots, s_n$ over some alphabet Σ, each of length L, and a number d. The question is whether there exists a string $t \in \Sigma^L$ that is at Hamming distance (i.e. number of differing symbols) at most d from each of the strings $s_1, \ldots, s_n$. Prove that this problem can be solved in time $(2d)^d \cdot (nL)^{\mathcal{O}(1)}$.