Parameterized algorithms — homework 6

Lower bounds, deadline: January 27th, 2019, 23:59

Problem 1. In the 2VC-Dimension problem we are given a graph G and an integer k. The question is whether in G one can find a set X consisting of exactly k vertices satisfying the following property: for every pair of distinct vertices $u, u' \in X$, there exists a vertex v in G such that $N[v] \cap X = \{u, u'\}$ (here, $N[v]$ denotes the set consisting of v and all neighbors of v). Prove that this problem is $\mathsf{W}[1]$-hard when parameterized by k and, assuming ETH, does not admit an algorithm with running time $f(k) \cdot \|G\|^{o(k)}$ for any computable function f.

Problem 2. In the Bounded Degree Vertex Deletion problem we are given a graph G and integers k and ℓ, and the question is whether one can find a set X consisting of at most k vertices of G such that in $G - X$ every vertex has degree at most ℓ. Prove that, assuming ETH, this problem cannot be solved in time $2^{\mathcal{O}(k \cdot f(\ell))} \cdot \|G\|^{\mathcal{O}(1)}$ for any function $f(\ell) \in o(\log \ell)$.

Problem 3. In the Directed Arc Multicut problem we are given a directed graph D, an integer k, and a sequence of requests $(s_1, t_1), \ldots, (s_\ell, t_\ell)$, where s_i-s and t_i-s are vertices of D. The question is whether one can remove at most k arcs from D so that for every $i \in \{1, \ldots, \ell\}$, the vertex t_i becomes not reachable from s_i. Prove that this problem is $\mathsf{W}[1]$-hard when parameterized by k only (i.e., ℓ can be unbounded).