Parameterized algorithms — homework 5

Cut problems and representative sets (**UPDATED**), deadline: January 18th, 2019

Problem 1. Let G be a graph and let s and t be two different vertices of G. For $X \subseteq V(G)$, let $\delta(X)$ denote the number of edges of G with one endpoint in X and second outside of X. Let k be the minimum size of a cut between s and t in G, that is, the minimum of $\delta(X)$ among X containing s and not containing t. Let

$$\mathcal{F} = \{Z :\ s \in Z,\ t \notin Z,\ \delta(Z) \leqslant k+1,\ \text{and } Z \text{ is inclusion-wise maximal subject to these conditions}\}.$$

Prove that $\mathcal{F}$ is a sunflower with pairwise non-adjacent petals; that is, there exists $C \subseteq V(G)$ such that for all different $Z, Z' \in \mathcal{F}$, we have $Z \cap Z' = C$ and there is no edge between $Z \setminus C$ and $Z' \setminus C$.

Problem 2. In the Subset Feedback Edge Set problem we are given a graph G together with a subset of edges S and an integer k. The question is whether one can remove at most k edges from G so that no cycle passing through an edge of S remains (note that edges of S can be also removed). Prove that this problem is fixed-parameter tractable when parameterized by $|S|$.

Problem 3. Consider the following variant of Multilinear Monomial Testing. We are given an arithmetic circuit C with p input gates labelled with variables $x_1, x_2, \ldots, x_p$ and one output gate, and an integer k. There are two types of gates in C: addition gates and multiplication gates, each with fan-in 2; note that we do not allow subtraction or negation gates. Let $P(x_1, \ldots, x_p) \in \mathbb{Z}[x_1, \ldots, x_p]$ be the polynomial over variables $x_1, x_2, \ldots, x_p$ computed by the circuit C. The question in the problem is the following: determine whether P, expanded into monomials, contains a multilinear monomial of degree k (i.e. monomial of the form $x_{i_1} x_{i_2} \ldots x_{i_k}$ for $i_1, \ldots, i_k$ pairwise distinct), and, provided this is the case, report any such monomial. Prove that this problem can be solved in deterministic time $2^{\mathcal{O}(k)} \cdot n^{\mathcal{O}(1)}$ where n is the size of C.