Parameterized algorithms — homework 4

Algebraic Techniques, deadline: 15th December 2018, 23:59

Problem 1. Given an n-vertex graph G and an integer k decide whether the vertices of G can be partitioned into k subsets $V_1, \ldots, V_k$, such that each of the induced graphs $G[V_i]$, $i = 1, \ldots, k$, is a forest. Show that the problem is solvable in time $2^n n^{O(1)}$.

Algorithms running in time $c^n n^{O(1)}$ for $c > 2$ are worth at most 3 points. Deterministic algorithms are graded better.

Problem 2. In a graph G with colored edges, a path P is *rainbow* if all its edges are of pairwise different colors. In the 2-Rainbow Connection problem we are given a graph G, and k pairs of vertices $(s_1, t_1), \ldots, (s_k, t_k)$. The question is whether there exists a coloring of edges of the given n-vertex G with 2 colors so that for every $i \in \{1, \ldots, k\}$, there exists a rainbow path connecting s_i with t_i. Show an algorithm running in time $2^k n^{O(1)}$. Your algorithm should be deterministic and use only polymomial space.

Note: slower algorithms do not get any points.

Problem 3. We are given three disjoint sets of vertices A, B and C such that $|A| = |B| = |C| = n$ and a set of triples $E \subseteq A \times B \times C$. We say that a subset $M \subset E$ is a *3-dimensional matching* when the triples in M are pairwise disjoint. The goal is to decide if there is a 3-dimensional matching M that matches all elemenets in A, B and C, i.e., $\bigcup_{T \in M} T = A \cup B \cup C$. Give a randomized algorithm that solves this problem in time $2^n n^{O(1)}$.

Algorithms running in time $c^n n^{O(1)}$ for $c > 2$ are worth at most 3 points.