Parameterized algorithms — homework 3

Treewidth, deadline: November 30th, 2018, 10:15 CET

Problem 1. In the Minimum Bisection problem we are given a graph G on $2n$ vertices and the task is to partition the vertex set of G into two subsets A and B, each of size n, so that the number of vertices with one endpoint in A and second in B is minimized. Prove that if G is given together with a tree decomposition of width at most t, then this problem can be solved in time $2^t \cdot n^{\mathcal{O}(1)}$.

Note: This result is stated in the Platypus book, but not proved there. Of course, it is not allowed to just refer to the book.

Problem 2. A subset of vertices $S \subseteq V(G)$ is *well-linked* if for all disjoint $A, B \subseteq S$ there are $\min(|A|, |B|)$ vertex-disjoint paths in G with one endpoint in A and second in B. Let $\mathbf{wl}(G)$ be the largest size of a well-linked set in G. Prove that there exist positive constants α, β such that $\alpha \cdot \mathbf{tw}(G) \leqslant \mathbf{wl}(G) \leqslant \beta \cdot \mathbf{tw}(G)$ for every graph G.

Problem 3. Let $\varepsilon > 0$ be a fixed real. Prove that given a planar graph G on n vertices and parameter k, one can in time $n^{\mathcal{O}(\varepsilon^{-2})}$ enumerate a family $\mathcal{F}$ of subsets of the vertex set of G with the following properties:

- $|\mathcal{F}| \leqslant n^{\mathcal{O}(\varepsilon^{-2})}$;
- for each $A \in \mathcal{F}$, the graph $G[A]$ has treewidth $\mathcal{O}(\varepsilon k)$; and
- for each $X \subseteq V(G)$ such that $|X| \leqslant k$ and $G[X]$ is connected, there exists $A \in \mathcal{F}$ such that $X \subseteq A$.

Note: During Tutorial 6 a stronger, and far more difficult result of this kind was stated. Of course, it is not allowed to just refer to this result.