Parameterized algorithms — homework 2

Iterative compression and color coding, deadline: **November 20th, 2018, 10:15 CET**

Note: In each of the problems, giving a randomized algorithm with the requested running time is worth 5 *points.*

Problem 1. In the Colorful Graph Motif problem we are given a graph G with vertices colored with k colors (this coloring is not necessarily proper). The question is whether there exists a connected subgraph of G that contains one vertex of each color. Prove that this problem can be solved in time $2^k \cdot \|G\|^{\mathcal{O}(1)}$.

Note: an algorithm with running time $3^k \cdot \|G\|^{\mathcal{O}(1)}$ *is worth* 5 *points.*

Problem 2. In a graph G with edges colored with r colors, a path P is *rainbow* if all its edges are of pairwise different colors. In the Rainbow Connection problem we are given a graph G, k pairs of vertices $(s_1, t_1), \ldots, (s_k, t_k)$, and a number r. The question is whether there exists a coloring of edges of G with r colors so that for every $i \in \{1, \ldots, k\}$, there exists a rainbow path connecting s_i with t_i. Prove that this problem is fixed-parameter tractable when parameterized by k and r.

Problem 3. In the Colorful Triangle Hitting problem we are given a graph G whose vertices are colored using k colors (this coloring is not necessarily proper). The question is whether one can select one vertex of each color so that every triangle (i.e. clique on 3 vertices) in G intersects the set of selected vertices. Prove that this problem can be solved in time $2^k \cdot \|G\|^{\mathcal{O}(1)}$.