Parameterized algorithms — homework 1

Branching and basic kernelization, deadline: October 30th, 2018, 10:15 CET

Problem 1. For an ordering $\sigma = (v_1, \ldots, v_n)$ of the vertices of a directed graph D, the *cost* of σ is $\sum_{(v_i,v_j)\in E(D)} \max(0, i-j)$. In the Optimum Linear Arrangement problem we are given a directed graph D and an integer $k \in \mathbb{N}$, and we ask whether there exists an ordering of the vertices of D with cost at most k. Prove that this problem parameterized by k has a kernel with $\mathcal{O}(k)$ vertices.

Bonus: You can get $+2$ *points by giving a kernel with at most* $(1+\varepsilon)k$ *vertices, for every* $\varepsilon > 0$.

Problem 2. A connected, undirected graph is a *caterpillar* if it can be obtained by taking a path and attaching degree 1 vertices to internal vertices of the path. A *caterpillarium* is a disjoint union of caterpillars. In the Caterpillarium Vertex Deletion problem we are given a graph G and an integer k, and the question is whether one can remove at most k vertices from G in order to obtain a caterpillarium. Design an algorithm solving this problem in time $c^k \cdot \|G\|^{\mathcal{O}(1)}$, for some constant c.

Bonus: You can get $+2$ *points if* $c < 7$, *and* $+4$ *points if* $c < 5$. *These are non-additive and applicable if at least* 5 *points are awarded.*

Problem 3. Consider the following variant of the Closest String problem. Given a set S of n words, each of length m, over the binary alphabet $\{0,1\}$, and two integers k and d, determine whether there is a word w of length m such that

- w contains exactly k ones, and
- for each $s \in S$ we have $\mathcal{H}(w, s) \leqslant d$.

Here, $\mathcal{H}(x, y)$ is the Hamming distance, i.e., the number of positions in which x and y differ. Design an algorithm for this problem running in time $d^{\mathcal{O}(d)} \cdot (n+m)^{\mathcal{O}(1)}$.