Meta-algorithms on graphs — problem batch 3

Bounded expansion and fo, meta-kernelization, deadline: 26.05.2017, 10:15 CET

Problem 1. In the Independent Dominating Set problem we are given a graph G, and the task is to find the smallest vertex subset S such that S is both an independent set and a dominating set in G. Prove that this problem does not have the finite integer index (FII) property.

Problem 2. Let Σ be the signature of rooted unranked, unordered trees, that is, Σ consists of one binary predicate selecting ancestor-descendant pairs. Prove that for every fixed positive integer d and every fo formula $\varphi(x, y)$ over Σ with two free variables, there exists a finite set $\mathcal{F}_{d,\varphi(x,y)}$ of triples of the form $(\psi_1(x), \psi_2(y), h)$, where $\psi_1(x), \psi_2(y)$ are fo formulas over Σ with one free variable and $h \leqslant d$ is an integer, such that the following holds. For all trees T of depth at most d and all nodes a, b of T, we have that $T \models \phi(a, b)$ if and only if there exists $(\psi_1(x), \psi_2(y), h) \in \mathcal{F}_{d,\varphi(x,y)}$ such that $T \models \psi_1(a)$, $T \models \psi_2(b)$, and the least common ancestor of a and b in T is at depth h.

Problem 3. Let $\mathcal{C}$ be a class of bounded expansion and let r, k be fixed integers. Prove that given a graph $G \in \mathcal{C}$ one can construct a data structure that can answer the following queries: for given vertices $u_1, \ldots, u_k$ of G, is it true that there exists a vertex v satisfying $\mathrm{dist}(u_i, v) \leqslant r$ for all $i \in \{1, 2, \ldots, k\}$. The data structure should have the following parameters:

- The running time of constructing the data structure is $\mathcal{O}(n^c)$ for some universal constant c, independent of $\mathcal{C}$, r, and k.
- The running time of answering one query is $\mathcal{O}(1)$.

Constants hidden in the $\mathcal{O}(\cdot)$-notation may depend on $\mathcal{C}$, r, and k.

Note: 5 *points will be granted for achieving query time complexity* $\mathcal{O}(\log n)$ *instead of* $\mathcal{O}(1)$.