Meta-algorithms on graphs — problem batch 1

MSO, tree decompositions, deadline: 10.04.2017, 10:15 CET

Problem 1. In the Channel Assignment problem we are given a graph G where each edge e is labelled by a request $\rho(e)$, which is a positive integer. The goal is to determine the minimum number W for which there is an assignment $\lambda\colon V(G) \to \{1,\ldots,W\}$ such that for each edge uv of G we have $|\lambda(u) - \lambda(v)| \geqslant \rho(uv)$. Construct an algorithm that solves this problem in time $2^{\mathcal{O}(k\log(kL))} \cdot n$, where k is the treewidth of the input graph, n is the number of its vertices, and L is the maximum among $\rho(e)$ for e ranging over the edges.

Problem 2. We are given an mso_2 formula $\varphi(u)$ with one free vertex variable u, and a graph G with n vertices, given together with its tree decomposition of width at most k. Prove that one can compute the set of all vertices $u \in V(G)$ such that $G, u \models \varphi$ in time $f(\|\varphi\|, k) \cdot n$, for some computable function f.

Problem 3. A vertex subset $X \subseteq V(G)$ of a graph G is a *vertex cover* of G if every edge of G has at least one endpoint in X. We are given an mso_2 sentence φ, a graph G, and some its vertex cover X. Give an algorithm deciding whether $G \models \varphi$ in time $2^{2^{\mathrm{poly}(\|\varphi\|,|X|)}} \cdot n$, where n denotes the number of vertices of G.