MAG — exercise session 4

Cliquewidth

Problem 1. Give an mso_2 transduction that given a graph, outputs its decomposition into 2-connected components.

Problem 2. Prove that every clique expression can be made *clean* without increasing its width in the following sense: during the computation, whenever we add all possible edges between colors i and j, there are no such edges so far.

Problem 3. Prove that the following problem can be solved in time $f(k, \|\varphi\|) \cdot n$: given a k-expression constructing a graph G on n vertices, and an mso_1 formula $\varphi(X)$, find the cardinality of the smallest $X \subseteq V(G)$ such that $G, X \models \varphi$.

Definition 1. A *branch decomposition* of a finite set U is a tree T with all inner nodes of degree 3 and a bijection λ between elements of U and the leaves of U. Given a function μ mapping bipartitions of U to reals, the *μ-width* of (T, λ) is the maximum of values of μ among bipartitions induced by the edges of T. The *μ-width* of U is the minimum μ-width among its branch decompositions.

Definition 2. The *rankwidth* of a graph G is the μ-width of $V(G)$, where μ of a bipartition of $V(G)$ is defined as the rank over $\mathbb{F}_2$ of the adjacency matrix induced by the bipartition.

Definition 3. The *branchwidth* of a graph G is the μ-width of $E(G)$, where μ of a bipartition of $E(G)$ is defined as the number of vertices incident to edges both from the left and from the right side of the bipartition.

Problem 4. Prove that for any graph G, it holds that

$$\mathsf{rw}(G) \leqslant \mathsf{cw}(G) \leqslant 2^{\mathsf{rw}(G)+1}.$$

Problem 5. Prove that for any graph G, it holds that

$$\mathsf{rw}(G) \leqslant \mathsf{tw}(G) + 1.$$

Problem 6. For a graph G, let $\rho(G)$ be the largest number t such that $K_{t,t}$ is not a subgraph of G. Prove that there is a function f such that

$$\mathsf{tw}(G) \leqslant f(\mathsf{cw}(G), \rho(G))$$

for every graph G.

Problem 7. Prove that for every graph G it holds that

$$\mathsf{bw}(G) - 1 \leqslant \mathsf{tw}(G) \leqslant \frac{3}{2}\mathsf{bw}(G).$$