Proportionality Guarantees in Elections with Interdependent Issues

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In an election on some (not necessarily binary) interdependent issues…

…some voters may be able to express their opinion using approval ballots…

Pizza? Burger? Yummy!
I do like donuts and choco-bars!
Don't you see that I am too young for alcohol?

…but others may not; mainly due to dependencies between issues!

I'd like a burger but only if paired with a beer.
I don't have a strong opinion when it comes to desserts.
I can't have both crab and chocolate!

[BL16]: framework for expressing dependencies in the context of AV

[SG22]: proportionality criterion for binary and independent issues & powerful guarantees for PAV and MES

For every sufficiently large group of voters, independent of cohesiveness requirements.

Any group of voters that makes up an x-fraction of the electorate, should be able to decide on an x-fraction of the issues.

Wow! Combining [BL16] with [SG22] might lead to provable proportionality guarantees for elections with interdependent issues!

Consider an election with n voters and m issues, each of domain d, s.t. voter v_i casts *dependency graph* G_i and *conditional approval ballots*.

$\mathcal{G}$:= undirected variant of $\bigcup_{\text{voter } i} G_i$

Δ := maximum degree in $\mathcal{G}$

$u_i(w)$:= satisfaction of voter v_i, under outcome $w \in d^m$

For a set of voters V, r_V := # issues: every $v_i \in V$ approves ≥ 1 alternative.

A rule R is *a-proportional*, $a \in [0,1]$, if for every set of voters V, there exists a voter $v_i \in V$ s.t. if w: winning outcome under R, then $u_i(w) > \alpha\, r_V \frac{|V|}{n} - 1$, for any instance.

Conditional Proportional Approval Voting (CPAV)

$$\textit{winning outcome} := argmax_{w \in d^m} \sum_{v_i \in N} \sum_{k=1}^{u_i(w)} 1/k$$

NP-hard

Conditional Method of Equal Shares (cMES)

- ★ assign a budget of m to every voter
- ★ for every yet unfixed issue I_j and for every possible (sub)outcome w for some k issues in the closed neighborhood of I_j in $\mathcal{G}$:
- ★ S(w) = voters with >0 budget, satisfied w.r.t. I_j under w.
- ★ p(w) = price s.t. if every voter in S(w) paid p(w) or all the money she has left, then voters from S(w) would altogether pay nk.
- ★ if no purchase can be made, fix remaining issues arbitrarily, otherwise, select w that minimizes p(w) and reduce voters' budget accordingly. Then, repeat from Step 2

observation 1. Any "reasonably fair" rule, cannot be α-proportional for $\alpha > {}^{1}/_{d^{\Delta+1}}$.

Assumption 1: Voters that can be satisfied w.r.t. to an issue, cannot be satisfied w.r.t. to other 'nearby' issues in $\mathcal{G}$.

theorem 2. Under Assumption 1, cPAV is α-proportional, for $\alpha = {}^{1}/_{(1+\Delta^2)\, d^{\Delta+1}}$.

Δ=0: α=1/d, strict generalization of the ½ factor (binary issues) [SG22] (tight according to Observation 1)

observation 3. For Δ>0, cMES cannot be α-proportional, for any α, even under Assumption 1.

cMES strictly worse than cPAV, in certain instances (in contrast to the Δ=0 case)

Assumption 2: For every voter v_i, for every issue I_j and for every combination of alternatives for issues in the in-neighborhood of I_j in $\mathcal{G}$, there is an alternative of I_j that satisfies v_i.

theorem 4. Under Assumption 2, cMES is α-proportional, for $\alpha = {}^{1}/_{(1+\Delta)\, d^{\Delta+1}}$.

Δ=0: α=1/d, strict generalization of the ½ factor (binary issues) [SG22] (tight according to Observation 1)

cMES NP-hard *even for binary domains, either for $\Delta \in \omega(1)$, or different G_i's.*

Contrasts the polynomial solvability of (unconditional) MES.

cMES ∈ P *for $\Delta \in O(1)$ and common G_i's.*

REFS

[BL16]: N. Barrot & J. Lang. Conditional and Sequential Approval Voting on Combinatorial Domains. In *International Joint Conference on AI (IJCAI)*, 2016.

[SG22]: P. Skowron & A. Gorecki. Proportional Public Decisions. In *AAAI Conference on AI* (AAAI), 2022.