# Delegation: A Small Step for a Voter, a Giant[1] Leap for Democracy

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[1]Ok, not always, I admit it, but I couldn't resist a clickbait title, sorry. By the end of the article, it will have become clear to you when and under which conditions the leap is (nearly) "giant" and when it's not.

[2]If you're wondering, I'd cast my vote for the lemonade, given the chance.

You can find more about Participatory Budgeting in $\Delta_{22}^{11}$ and $\Delta_{24}^2$.

What a pleasant surprise: our municipality's treasury is overflowing with funds. Alright, what's the plan with all that cash? I've heard a few ideas floating around: How about building a community composting facility, setting up public Wi-Fi hotspots, installing interactive art installations throughout the town, or treating everyone to free lemonade during those scorching summer days? It would be wonderful if citizens had the opportunity to decide.[2]

Every year, a portion of public funds is assigned to various projects through Participatory Budgeting procedures. Let's take a step back and examine the stage preceding the voting process to understand how the pool of proposals, from which each voter selects their preferences, could have been formed. One way would be for authorities to suggest a couple of projects, and another is to encourage citizens to submit their own project ideas. The second method sounds like the better choice, but here is a crucial concern: If authorities only suggest a handful of projects, each participant could probably find some time to thoroughly evaluate each one and voice their opinion, but does the same hold if every participant submits their own proposal, or even multiple proposals? Unfortunately, among a huge pool of proposals, the only practical solution for a voter would be to review only a subset of the available choices, resulting to an electorate consisting of voters whose preferences are incomplete. Actually, incomplete preferences are pretty common in our daily lives: Who has the time to watch a trailer for every single movie on Netflix before picking one? Or to read reviews for every street food joint in their new neighborhood before settling on dinner? Or to explore every available field of study before choosing a university major? In general, this might not be tremendously bad for certain everyday decisions. However, when it comes to elections, incomplete preferences can lead to the selection of proposals that fall far short of the optimal one–the choice voters would have made if they had complete preferences. Alright, let's see if I can read your mind... Yep, got it! You were hoping for an example to clear things up, weren't you? Well, here it is.

**Incomplete Preferences at Play: Mr. Voter's and Mrs. Voterina's Votes.** Consider a scenario where Mr. Voter and Mrs. Voterina must select 1 among 3 projects and let's say that each project is pretty tricky to form an opinion about. So, due to time constraints, each voter is only capable of expressing an opinion about a subset of the projects. Say for instance that both voters form an opinion only on projects 1 and 2. Let's say Mr. Voter favors the first project and dislikes the second, while Mrs. Voterina accepts the second and dislikes the first. Then, each of the first two projects has exactly one supporter, and any fair rule would arbitrarily select one. Now, let's imagine we magically give the voters ample time to review all the proposals. If this were the case, say that they would both also vote in favor of proposal 3. But alas, they don't have that luxury of time (this is why their preferences regarding project 3 appear in gray background in the side table). Do you see the issue? The chosen proposal has only one supporter, yet there exists another proposal that would have garnered the support of all voters if time permitted, nevertheless, due to the limited time for each voter in the election process, it did not win. And ok, a winning proposal with 1 supporter in an instance where the optimal approval score is 2 isn't that bad; however, it's possible to generalize such an instance towards ending up electing a proposal with only 1 approval in an instance where the optimal (given voters' intrinsic preferences) would have had n approvals. In summary, the initial idea of allowing voters to propose projects seemed promising, but, this resulted in a plethora of candidate proposals, leading to incomplete preferences and in turn potentially unfavorable outcomes. Reflecting on the scenario of Mr. Voter and Mrs. Voterina, Professor Ballot wonders: *"Hmm, incomplete preferences seem like a problem, but what can we even do about these?"*

|         | Mr. Voter | Mrs. Voterina |
|---------|-----------|---------------|
| proj. 1 | yes       | no            |
| proj. 2 | no        | yes           |
| proj. 3 | yes       | yes           |

**1**

## Liquid Democracy

Back in our school days, we learned about two types of democracy: direct and representative. Sometimes, trying to juggle two watermelons in one hand doesn't end well, but that's not always the case. For instance, I remember a day when I managed to pull it off; okay, they were two quite small watermelons, but that's beside the point (as is this sentence!). Let's just stick to the two types of democracy and consider, what if, instead of selecting one mode of participation and following it for everyone, we let voters choose how they want to get involved? Picture this: voters could opt for direct involvement (akin to direct democracy) when they feel well-informed or when they disagree with all available representatives. Alternatively, they could entrust their vote to a proxy (like in representative democracy) based on trust and shared beliefs. In the end, a voter who decides to cast a ballot does so with a weight based on the number of agents they represent. This hybrid voting system is called Liquid Democracy. Fun(?) Fact: In certain contexts, transitive delegations are being permitted, which allows a delegate's vote being passed on again to the proxy's proxy, and so forth.

We assume that proxies have good intentions, know the full voters' true preferences, have no personal preferences and are capable of breaking ties based on their likes. It sounds quite unrealistic, right? Well, this optimistic best-case scenario serves a purpose: to highlight the limitations of proxy voting. If a good solution can't be guaranteed with such an ideal proxy, then it will not be achievable in any case. On the other hand, regarding positive results, they establish the limits of the aforementioned impossibilities. Specifically, if something is not deemed impossible in the studied setting or if a certain restriction of the setting allows for positive results, then this should be the starting point for a work on a more realistic model. Of course one should expect slightly less favorable guarantees under the same conditions in a scenario where proxies are not as knowledgeable and well-intentioned as described, or it might require a few more proxies, a more relaxed agreement threshold, or a more structured instance to achieve the same guarantees.

The studied model is inspired from real-world implementations of Participatory Budgeting in blockchain governance (e.g., Project Catalyst). In such systems, public funds are regularly being allocated according to the preferences of the stakeholders of the associated cryptocurrency. Community members can propose ideas for spending the public budget through submitted proposals. Stakeholders then have the choice to vote directly for or against these proposals, or delegate their voting power to proxies. The proxies could be seen as experts tasked with thoroughly evaluating the proposed ideas to help elect socially beneficial proposals.

| | Ballot Villain | $n-1$ voters |
|---|---|---|
| proj. 1 | yes | no |
| proj. 2 | no | yes |
| proj. 3 | no | yes |
| proj. 4 | yes | yes |

**Incomplete Votes? Fear Not, Proxy Voting is Here to (try to) Save the Day.** So, where were we? Ah, yes, elections with incomplete preferences! Let's focus on a scenario where voters can either approve or disapprove each candidate project, starting with the simplest case of electing just a single project. With the preceding discussion on Liquid Democracy in mind, you might see where I'm headed with this: a voter's opinions may not cover all proposals but delegating their ballot to a trusted proxy could lead to a better outcome (Spoiler Alert: or it might not, in certain cases). Each voter who wishes to delegate selects a proxy based on the alignment of their preferences regarding the proposals for which the voter has opinions on. More precisely, each voter sets an agreement threshold for delegation, and if a proxy's ballot aligns with the voter's opinion on their revealed proposals to a degree that meets the specified threshold, the voter delegates their ballot to that proxy. For simplicity, let's assume this threshold is half of the revealed issues for each voter (which is that each voter may opt to delegate if they find a proxy who shares their views on at least half of the projects they've formed opinions on).

**Mr. Voter, Mrs. Voterina and their Electorate Hero: A Story of Success.** Did the previous paragraph seem overly technical? I'm glad you didn't turn the page. Let's simplify things by revisiting the example of Mr. Voter and Mrs. Voterina, this time with the addition of a proxy voter. Imagine that a wild wise proxy appears out of nowhere to assist Mr. Voter and Mrs. Voterina in their decision-making process. We'll call this proxy Electorate Hero. Suppose that the Electorate Hero announces that they'll vote in favor of all 3 proposals. Both Mr. Voter and Mrs. Voterina agree with the proxy on half of the proposals they are informed about, leading both of them to delegate their ballots. As a result, the proxy becomes the sole voter. With a tie-breaking rule decided by the Electorate Hero, aligned with voters' intrinsic preferences, the optimal proposal, project 3, is elected. So, by introducing a proxy, we've achieved the best possible outcome! Pretty neat, huh?

Professor Ballot appears again, this time reformulating his previous question:

**The Big Question:** *Could potential proxies come to the rescue in elections of incomplete votes, towards helping in electing an outcome that closely aligns with the optimal solution?*

**The Big Question (and a small oops in the first attempt to address it).** Does the Big Question of Professor Ballot simply come down to finding what to advertise in order to attract multiple voters? At first glance, it may seem that way, but what we're overlooking here is that the proxy doesn't just use their ballot to attract voters and increase their voting power; they also cast this ballot. Therefore, this advertisement should be even more carefully selected. The example illustrated in the margin, which involves $n$ voters (one of whom will be referred to as the Ballot Villain, a naming that will soon become evident) and 4 proposals, highlights this distinction. Suppose all voters have only formed concrete preferences regarding the first 3 projects (and not the last one). The voters' opinions on the ultimate project (appearing in gray background) are unknown to them (but known to the proxy, who perhaps leverages historical voting patterns on similar projects

to conduct an analysis and infer the voters' opinions on these). Say a proxy advertises that they will vote in favor of all proposals. Indeed (while it may not be the best option) it is a ballot that manages to draw in multiple voters. The plan here is to demonstrate that this appeal alone isn't enough to ensure the selection of a quality proposal. Specifically, the Ballot Villain, agrees with only one out of three proposals that they are aware of alongside the proxy; they perceive this as falling below the agreement threshold for delegation (set to be half of the projects for which they have formed an opinion) and decides not to delegate but vote directly. On the contrary, the remaining $n-1$ voters do delegate, as there is a commonality in ballots that comprises over half of the total number of projects they have reviewed. As a result, the winning proposal will be the first one, as it is approved by both the first voter and the proxy, who holds all the remaining voting power; all other proposals only have approval from the proxy. Therefore, despite attracting almost all voters, introducing such a proxy could lead to choosing a (far) less-than-ideal proposal. Interestingly, in this example, without the proxy, direct voting would result in a winning proposal approved by $n-1$ voters. Well, the introduction of (such an arbitrary) proxy didn't quite go as planned. I hope that it is not only me who thinks that this example is fascinating, revealing that the problem is more complex and intriguing than we initially thought.

### There's Some Good in the Elections of Incomplete Preferences, Mr. ~~Frodo~~ Voter, and (with the Aid of Proxies) it's Worth Fighting for.

In our work ([1]) we showed a rather strong negative result: that there exist situations the addition of proxies cannot lead to favorable outcomes at all! Nonetheless, there's hope on the horizon. We're sorry about the bad instances, but not only they present us the limitations of proxy voting in settings with incomplete preferences but also they're helping us identify conditions for positive results. And guess what? We've found some! For instance, if all voters reviewed exactly the same set of proposals, a proxy can advertise their ballot in a way that ensures the elected solution has an approval score no less than the optimal approval score divided by 3. Now, you might be wondering how a number appeared here and if there's any room for improvement. Explaining the first part would take us on a bit of a journey through technicalities and quite long explanations, so let's just skip over it. As for the second part, well, there's a chance things could get better, but I'm pretty sure we won't top the optimal divided by anything less than 1.6. And now you're probably curious about where a number came from this time. Once again, I won't dive into that, but it's cute that we have it, isn't it?

Here's a more surprising result: adding a second proxy can always lead to the optimal solution. How? Easy! One proxy advertises an approval towards all projects, while the other advertises an approval only towards the voters' favorite project (and a disapproval for all the rest). Suppose that, as assumed, a voter delegates if they agree with the proxy in at least half the proposals included in their ballot. Then each voter will delegate to one of the proxies. (Do you see why? In high level, it is because each voter either has at least half "yes" or half "no" responses in their ballot.) Fortunately, the proxies, together, will elect the optimal proposal as it is the only approved by both of them. We conclude with another result that could be seen as a pleasant bottom line: there's a bounded approximation guarantee that hinges on the thresholds for voters' delegation, the commonalities of their revealed proposal sets, and the number of proxies involved.

And now, as a reward for sticking with me until now, here's a little gift for you: I'd like to bring to your attention that there are still plenty of open avenues for further exploration! (Alright, it might not be the most thrilling gift you've ever received, but hey, what were you expecting?) Examining the tightness of the results, delving into other information or rationality models for the proxies, as well as working with different distance metrics, voting rules, or settings with multiple winners, are all open problems that might be waiting for you to tackle!

[1] Georgios Amanatidis, Aris Filos-Ratsikas, Philip Lazos, Evangelos Markakis, and Georgios Papasotiropoulos. On the Potential and Limitations of Proxy Voting: Delegation with Incomplete Votes. *Proceedings of the '23rd International Conference on Autonomous Agents and Multiagent Systems*, 2024.