Homework assignment for 23rd March

March 16, 2017

Some of the problems below were solved in the class.

Exercise 1. Let M be the following Kripke model for intuitionistic logic. M has infinitely many worlds w_n for $n \in \mathbb{N}$ (here $0 \in \mathbb{N}$). For each n, the domain of w_n is $\mathbb{N}$. A model w_i is accessible from w_j if and only if $i \geq j$. The signature consists of one unary predicate $P(x)$ such that in the world w_n, $P(k)$ holds iff $k \leq n$. Check whether the following sentences are satisfied in M:

- $\exists x\ P(x)$.
- $\neg\forall x \neg P(x)$.
- $\neg\exists x \neg P(x)$.
- $\forall x \exists y\ (P(x) \to \neg P(y))$.
- $\exists x (P(x) \to \forall y\ P(y))$.

Exercise 2. Let M be the following Kripke model for the intuitionistic logic. M has infinitely many worlds w_n for $n \in \mathbb{N}$. For each n, the domain of w_n is $\{0, 1, \ldots, n\}$. A model w_i is accessible from w_j if and only if $i \geq j$. The signature consists of one unary predicate $P(x)$ such that in the world w_n, $P(k)$ holds iff $k < n$ (note that in each world there is exactly one element x which does not satisfy $P(x)$). Check whether the following sentences are satisfied in M:

- $\neg\forall x P(x)$.
- $\exists x \neg P(x)$.
- $\forall x \exists y\ \neg P(y)$.
- $\exists x (\exists y P(y) \to P(x)))$.

Exercise 3. Find a sentence which is true in exactly one of the models described in the above problems.

Exercise 4. For each of the following formulae, find a model in which they are not satisfied.

- $\neg\neg\forall x P(x) \to \exists x P(x)$.
- $\forall x \exists y (P(x) \vee \neg P(y))$.
- $\exists x \forall y (P(x) \to P(y))$.

Observe that all the above sentences are classically valid.

Exercise 5. Find a model in which only one of the following sentences holds:

- $\neg P(a) \vee Q(a)$.
- $P(a) \to Q(a)$.

Exercise 6. Find a model in which only one of the following sentences holds:

- $\forall x (\exists y Q(y) \vee P(x))$.
- $\exists y\, Q(y) \vee \forall x\, P(x)$