Homework assignment for 15th October

8 października 2019

This assignment is due on 15th October. You will declare in the class which problems you solved. You do not have to prepare written solutions, but you might be asked to present your solutions in the class

Exercise 1. We define **Scheffer stroke** as a binary propositional connective | such that $\phi|\psi$ is true iff neither ϕ nor ψ is true. Show that any binary propositional connective can be defined using Scheffer symbol.

Exercise 2. Show that not every binary propositional symbol can be defined using the equivalence symbol $\Leftrightarrow$.

Exercise 3. Fix a set of variables A. Suppose that π is a formula in the disjunctive normal form such that $v \models \pi$ iff the number of variables p such that $v(p) = 1$ is even. Show that the number of symbols in π is at least 2^n.

Problems solved in today's class – 8th October.

Exercise 1. Is this formula a tautology:

- $(p \to q) \vee (q \to p)$.

- $\Big((p \Leftrightarrow q) \Leftrightarrow r\Big) \Leftrightarrow \Big(p \Leftrightarrow (q \Leftrightarrow r)\Big)$.

- $\Big(p \to ((q \vee r) \wedge s)\Big) \vee \Big(s \to (q \wedge p)\Big)$.

Exercise 2. Is this formula a tautology:

- $\bigwedge_{i=1}^{n+1} \bigvee_{j=1}^{n} p_{i,j} \to \bigvee_{i=1}^{n+1} \bigvee_{j<k=2}^{n} p_{i,j} \wedge p_{i,k}$.

- $\bigwedge_{i=1}^{n+1} \bigvee_{j=1}^{n} p_{i,j} \to \bigvee_{j=1}^{n} \bigvee_{i_1<i_2=2}^{n+1} p_{i_1,j} \wedge p_{i_2,j}$.

Exercise 3. Show that for any propositional formula ϕ, there is a formula ϕ' equivalent to ϕ such that ϕ' is in the disjunctive normal form.

Exercise 4. Show that for any propositional formula ϕ, there is a formula ϕ' equivalent to ϕ such that ϕ' is in the conjunctive normal form.