

A = {{2, -1, 1, 2}, {1, 2, 3, 1}, {0, 0, 2, -1}, {0, 0, 1, 2}};

MatrixForm[A]

$$\begin{pmatrix} 2 & -1 & 1 & 2 \\ 1 & 2 & 3 & 1 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

chi = Det [A - t IdentityMatrix[4]]

Factor[chi]

Solve[chi == 0, t]

$$25 - 40 t + 26 t^2 - 8 t^3 + t^4$$

$$(5 - 4 t + t^2)^2$$

{{t → 2 - i}, {t → 2 - i}, {t → 2 + i}, {t → 2 + i}}

B = A - (2 + I) IdentityMatrix[4]; MatrixForm[B]

$$\begin{pmatrix} -i & -1 & 1 & 2 \\ 1 & -i & 3 & 1 \\ 0 & 0 & -i & -1 \\ 0 & 0 & 1 & -i \end{pmatrix}$$

NullSpace[B]

K2 = NullSpace[B.B]

w2 = K2[[1]]

w1 = B.w2

{{i, 1, 0, 0}}

{{-5 i, 0, 2 i, 2}, {i, 1, 0, 0}}

{-5 i, 0, 2 i, 2}

{-1 + 2 i, 2 + i, 0, 0}

v1 = Conjugate[w1]

v2 = Conjugate[w2]

{-1 - 2 i, 2 - i, 0, 0}

{5 i, 0, -2 i, 2}

(* Macierz przejcia z bazy Jordana do standardowej *)

J = Transpose[{v1, v2, w1, w2}]; MatrixForm[J]

$$\begin{pmatrix} -1 - 2 i & 5 i & -1 + 2 i & -5 i \\ 2 - i & 0 & 2 + i & 0 \\ 0 & -2 i & 0 & 2 i \\ 0 & 2 & 0 & 2 \end{pmatrix}$$

MatrixForm[Inverse[J].A.J]

$$\begin{pmatrix} 2 - i & 1 & 0 & 0 \\ 0 & 2 - i & 0 & 0 \\ 0 & 0 & 2 + i & 1 \\ 0 & 0 & 0 & 2 + i \end{pmatrix}$$

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(* baza rzeczywista *)  
a1 = (v1 + w1) / 2  
a2 = (v2 + w2) / 2  
b1 = (v1 - w1) / (2 I)  
b2 = (v2 - w2) / (2 I)  
  
{-1, 2, 0, 0}  
  
{0, 0, 0, 2}  
  
{-2, -1, 0, 0}  
  
{5, 0, -2, 0}  
  
JJ = Transpose[{a1, b1, a2, b2}]  
  
MatrixForm[Inverse[JJ].A.JJ]  
{{-1, -2, 0, 5}, {2, -1, 0, 0}, {0, 0, 0, -2}, {0, 0, 2, 0}}
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$$\begin{pmatrix} 2 & -1 & 1 & 0 \\ 1 & 2 & 0 & 1 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$