In[1]:= `ker[A_] := Module[{X = NullSpace[A]}, Print["ker ", MatrixForm[X]]; X]`

In[2]:= `A = {{0, 0, 0, 0, 0, 6, 0}, {4, 4, 0, 3, 0, -4, 0}, {0, 0, 4, 0, 0, 0, 2}, {0, 0, 0, 4, 0, 0, 0}, {4, 2, 0, 2, 4, -4, 0}, {-4, 0, 0, -1, 0, 10, 0}, {4, 0, 0, 5, 0, -4, 4}};`
`MatrixForm[`
`A]`

Out[3]//MatrixForm=

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 6 & 0 \\ 4 & 4 & 0 & 3 & 0 & -4 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 4 & 2 & 0 & 2 & 4 & -4 & 0 \\ -4 & 0 & 0 & -1 & 0 & 10 & 0 \\ 4 & 0 & 0 & 5 & 0 & -4 & 4 \end{pmatrix}$$

In[4]:= `(* wielomian charakterystyczny *)`
`chi[t] = Det[A - t IdentityMatrix[7]]`

Out[4]= $24576 - 40960\,t + 29184\,t^2 - 11520\,t^3 + 2720\,t^4 - 384\,t^5 + 30\,t^6 - t^7$

In[5]:= `Factor[chi[t]]`

Out[5]= $-(-6 + t)\,(-4 + t)^6$

In[6]:= `(* wartosc wlasna 6 *)`
`B6 = A - 6 IdentityMatrix[7]; MatrixForm[B6]`
`K = ker[B6];`

Out[6]//MatrixForm=

$$\begin{pmatrix} -6 & 0 & 0 & 0 & 0 & 6 & 0 \\ 4 & -2 & 0 & 3 & 0 & -4 & 0 \\ 0 & 0 & -2 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & -2 & 0 & 0 & 0 \\ 4 & 2 & 0 & 2 & -2 & -4 & 0 \\ -4 & 0 & 0 & -1 & 0 & 4 & 0 \\ 4 & 0 & 0 & 5 & 0 & -4 & -2 \end{pmatrix}$$

ker $\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$

In[8]:= `u1 = K[[1]]`

Out[8]= {1, 0, 0, 0, 0, 1, 0}

In[9]:= `(* wartosc wlasna 4 *)`
`B4 = A - 4 IdentityMatrix[7]; MatrixForm[B4]`
`K1 = ker[B4];`

Out[9]//MatrixForm=

$$\begin{pmatrix} -4 & 0 & 0 & 0 & 0 & 6 & 0 \\ 4 & 0 & 0 & 3 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 0 & 2 & 0 & -4 & 0 \\ -4 & 0 & 0 & -1 & 0 & 6 & 0 \\ 4 & 0 & 0 & 5 & 0 & -4 & 0 \end{pmatrix}$$

ker $\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$

In[11]:=
```
MatrixForm[B4.B4]
K2 = ker[B4.B4];
```

Out[11]//MatrixForm=

$$\begin{pmatrix} -8 & 0 & 0 & -6 & 0 & 12 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 8 & 0 & 0 & 10 & 0 & -8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 10 & 0 & -8 & 0 \\ -8 & 0 & 0 & -6 & 0 & 12 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 \end{pmatrix}$$

$$\text{ker} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[13]:=
```
MatrixForm[B4.B4.B4]
K3 = ker[B4.B4.B4];
```

Out[13]//MatrixForm=

$$\begin{pmatrix} -16 & 0 & 0 & -12 & 0 & 24 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 & 0 & 0 \\ -16 & 0 & 0 & -12 & 0 & 24 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\text{ker} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 3 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[15]:=
```
MatrixForm[B4.B4.B4.B4]
K4 = ker[B4.B4.B4.B4];
```

Out[15]//MatrixForm=

$$\begin{pmatrix} -32 & 0 & 0 & -24 & 0 & 48 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -32 & 0 & 0 & -24 & 0 & 48 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\text{ker} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 3 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -3 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[17]:=
```
(* najdluzszy lancuszek *)
v4 = K4[[4]]
v3 = B4.v4
v2 = B4.B4.v4
v1 = B4.B4.B4.v4
B4.B4.B4.B4.v4
```

Out[17]= {-3, 0, 0, 4, 0, 0, 0}

Out[18]= {12, 0, 0, 0, -4, 8, 8}

Out[19]= {0, 16, 16, 0, 16, 0, 16}

Out[20]= {0, 0, 32, 0, 32, 0, 0}

Out[21]= {0, 0, 0, 0, 0, 0, 0}

In[22]:=
```
(* Z tych wektorow trzeba wybrac koncowy element drugiego lanscuszka,
aby wraz z v1 rozpinal K1. Ta metoda wymaga pozniej
 szukania przeciwobrazu v1 i nie jest bardzo wygodna*)
K1
```

Out[22]= {{0, 0, 0, 0, 1, 0, 0}, {0, 0, 1, 0, 0, 0, 0}}

In[23]:=
```
w1 = K1[[1]]
```

Out[23]= {0, 0, 0, 0, 1, 0, 0}

In[24]:=
```
(* zastanawiamy sie czego obrazem jest w1 *)
MatrixForm[B4]
```

Out[24]//MatrixForm=

$$\begin{pmatrix} -4 & 0 & 0 & 0 & 0 & 6 & 0 \\ 4 & 0 & 0 & 3 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 2 & 0 & 2 & 0 & -4 & 0 \\ -4 & 0 & 0 & -1 & 0 & 6 & 0 \\ 4 & 0 & 0 & 5 & 0 & -4 & 0 \end{pmatrix}$$

In[25]:=
```
w2 = {0, 1 / 2, 0, 0, 0, 0, 0};
B4.w2
```

Out[26]= {0, 0, 0, 0, 1, 0, 0}

In[27]:=
```
(* macierz przejscia z Jordana do standardowej *)
J = Transpose[{u1, v1, v2, v3, v4, w1, w2}];
MatrixForm[J]
Det[J]
```

Out[28]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 12 & -3 & 0 & 0 \\ 0 & 0 & 16 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 32 & 16 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 32 & 16 & -4 & 0 & 1 & 0 \\ 1 & 0 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 16 & 8 & 0 & 0 & 0 \end{pmatrix}$$

Out[29]= -4096

In[30]:=
```
(* postac Jordana *)
MatrixForm[Inverse[J].A.J]
```

Out[30]//MatrixForm=

$$\begin{pmatrix} 6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 \end{pmatrix}$$