In[1]:=

```
ker[A_] := Module[{X = NullSpace[A]}, Print["ker ", MatrixForm[X]]; X]
niezalezne[A_] := (Length[NullSpace[Transpose[A]]] == 0)
```

In[3]:=

```
(*A={{1,0,0,0,2,0,2,2,0,0},{0,1,0,0,0,0,0,0,0,0},
   {0,1,2,0,0,0,0,0,1,0},{1,0,1,1,1,0,0,2,0,1},{0,0,0,0,1,0,1,0,0,0},
   {0,1,1,0,0,1,0,0,1,0},{0,0,0,0,0,0,1,0,0,0},{0,1,0,0,0,0,0,2,1,1},
   {0,1,0,0,0,0,1,0,1,0},{0,0,0,0,0,0,1,0,0,1}};*)
A = {{3, -8, 0, 0, 2, 0, -6, -2, -4, -4}, {0, 3, 0, 0, 0, 0, 0, 0, 0, 0},
   {0, -3, 2, 0, 0, 0, -2, 0, -1, 0}, {1, -20, -1, 3, 1, 0, -18, -6, -10, -7},
   {0, 0, 0, 0, 3, 0, 1, 0, 0, 0}, {0, -3, -1, 0, 0, 3, -2, 0, -1, 0},
   {0, 0, 0, 0, 0, 0, 3, 0, 0, 0}, {0, -3, 0, 0, 0, 0, -4, 2, -1, -1},
   {0, 1, 0, 0, 0, 0, 1, 0, 3, 0}, {0, 0, 0, 0, 0, 0, 1, 0, 0, 3}};
MatrixForm[
 A]
```

Out[4]//MatrixForm=

3	−8	0	0	2	0	−6	−2	−4	−4
0	3	0	0	0	0	0	0	0	0
0	−3	2	0	0	0	−2	0	−1	0
1	−20	−1	3	1	0	−18	−6	−10	−7
0	0	0	0	3	0	1	0	0	0
0	−3	−1	0	0	3	−2	0	−1	0
0	0	0	0	0	0	3	0	0	0
0	−3	0	0	0	0	−4	2	−1	−1
0	1	0	0	0	0	1	0	3	0
0	0	0	0	0	0	1	0	0	3

In[5]:=

```
(* wielomian charakterystyczny *)
chi[t] = Det[A - t IdentityMatrix[10]]
```

Out[5]= $26\,244 - 96\,228\,t + 158\,193\,t^2 - 153\,576\,t^3 + 97\,524\,t^4 - 42\,336\,t^5 + 12\,726\,t^6 - 2616\,t^7 + 352\,t^8 - 28\,t^9 + t^{10}$

In[6]:=

```
Factor[chi[t]]
```

Out[6]= $(-3+t)^8\,(-2+t)^2$

In[7]:=

```
(* wartosc wlasna 2 *)
B2 = A - 2 IdentityMatrix[10]; MatrixForm[B2]
K = ker[B2];
```

Out[7]//MatrixForm=

1	−8	0	0	2	0	−6	−2	−4	−4
0	1	0	0	0	0	0	0	0	0
0	−3	0	0	0	0	−2	0	−1	0
1	−20	−1	1	1	0	−18	−6	−10	−7
0	0	0	0	1	0	1	0	0	0
0	−3	−1	0	0	1	−2	0	−1	0
0	0	0	0	0	0	1	0	0	0
0	−3	0	0	0	0	−4	0	−1	−1
0	1	0	0	0	0	1	0	1	0
0	0	0	0	0	0	1	0	0	1

ker $\begin{pmatrix} 2 & 0 & 0 & 4 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$

In[9]:=

```
(* dwie jednowymiarowe klatki Jordana *)
u1 = K[[1]]
u2 = K[[2]]
```

Out[9]= {2, 0, 0, 4, 0, 0, 0, 1, 0, 0}

Out[10]= {0, 0, 1, 1, 0, 1, 0, 0, 0, 0}

In[11]:=
```
(* wartosc wlasna 3 *)
B1 = A - 3 IdentityMatrix[10]; MatrixForm[B1]
K1 = ker[B1];
```

Out[11]//MatrixForm=

$$\begin{pmatrix} 0 & -8 & 0 & 0 & 2 & 0 & -6 & -2 & -4 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -3 & -1 & 0 & 0 & 0 & -2 & 0 & -1 & 0 \\ 1 & -20 & -1 & 0 & 1 & 0 & -18 & -6 & -10 & -7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -3 & -1 & 0 & 0 & 0 & -2 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 & 0 & 0 & -4 & -1 & -1 & -1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

ker $\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 1 \\ 2 & 0 & -1 & 0 & 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

In[13]:=
```
MatrixForm[B1.B1]
K2 = ker[B1.B1];
```

Out[13]//MatrixForm=

$$\begin{pmatrix} 0 & 2 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 3 & 1 & 0 & 2 & 0 & 4 & 4 & 3 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 2 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

ker $\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

In[15]:=
```
MatrixForm[B1.B1.B1]
K3 = ker[B1.B1.B1];
```

Out[15]//MatrixForm=

$$\begin{pmatrix} 0 & -4 & 0 & 0 & 0 & 0 & -4 & -2 & -2 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & -1 & 0 & 0 & 0 & -1 & 0 & -1 & 0 \\ 0 & -12 & -1 & 0 & 0 & 0 & -11 & -4 & -5 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & -1 & 0 & 0 & 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 & 0 & 0 & -2 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\text{ker} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[17]:=
```
MatrixForm[B1.B1.B1.B1]
K4 = ker[B1.B1.B1.B1];
```

Out[17]//MatrixForm=

$$\begin{pmatrix} 0 & 4 & 0 & 0 & 0 & 0 & 4 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 10 & 1 & 0 & 0 & 0 & 9 & 4 & 5 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 2 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\text{ker} \begin{pmatrix} 0 & -1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & -1 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[19]:=
```
(* z K4 trzeba wybrac poczatek najdluzszego lancuszka *)
K4 = ker[B1.B1.B1.B1];
```

$$\text{ker} \begin{pmatrix} 0 & -1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & -1 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[20]:=
```
v4 = K4[[1]]
v3 = B1.v4
v2 = B1.v3
v1 = B1.v2
B1.v1
```

Out[20]= {0, -1, 2, 0, 0, 0, 0, 0, 0, 2}

Out[21]= {0, 0, 1, 4, 0, 1, 0, 1, -1, 0}

Out[22]= {2, 0, 0, 3, 0, 0, 0, 0, 0, 0}

Out[23]= {0, 0, 0, 2, 0, 0, 0, 0, 0, 0}

Out[24]= {0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

In[25]:=
```
(* z K2 trzeba wybrac poczatek drugiego lancuszka *)
K2 = ker[B1.B1];
```

$$\text{ker}\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 & -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[26]:=
```
w2 = K2[[3]] (* 3 *)
niezalezne[{v1, v2, v3, v4, w2}]
```

Out[26]= {0, -1, 1, 0, -1, 0, 1, 0, 0, 0}

Out[27]= True

In[28]:=
```
w1 = B1.w2
niezalezne[{v1, v2, v3, v4, w1, w2}]
```

Out[28]= {0, 0, 0, 0, 1, 0, 0, -1, 0, 1}

Out[29]= True

In[30]:=
```
(* z K1 wybieramy pozostale 2 wektory bazy *)
K1 = ker[B1];
```

$$\text{ker}\begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 1 \\ 2 & 0 & -1 & 0 & 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

In[31]:=
```
x = K1[[2]] (* 2 *)
niezalezne[{x, v1, w1}]
```

Out[31]= {2, 0, -1, 0, 1, 0, 0, -1, 1, 0}

Out[32]= True

In[33]:=
```
y = K1[[3]]
niezalezne[{x, y, v1, w1}]
```

Out[33]= {0, 0, 0, 0, 0, 1, 0, 0, 0, 0}

Out[34]= True

In[35]:=
```
(* tego nie trzeba sprawdzac *)
niezalezne[{u1, u2, v1, v2, v3, v4, w1, w2, x, y}]
```

Out[35]= True

In[36]:=
```
(* macierz przejscia z bazy Jordana do standardowej *)
J = Transpose[{u1, u2, v1, v2, v3, v4, w1, w2, x, y}];
MatrixForm[J]
Det[J]
```

Out[37]//MatrixForm=

$$\begin{pmatrix} 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 2 & 0 & 1 & -1 & 0 \\ 4 & 1 & 2 & 3 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 \end{pmatrix}$$

Out[38]= 4

In[39]:=
```
(* postac Jordana *)
MatrixForm[Inverse[J].A.J]
```

Out[39]//MatrixForm=

$$\begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \end{pmatrix}$$

In[40]:=