Pure homology of algebraic varieties

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Abstract

We show that for a complete complex algebraic variety the pure term of the weight filtration in homology coincides with the image of intersection homology. Therefore pure homology is topologically invariant. To obtain slightly more general results we introduce *image homology* for noncomplete varieties.

KEY WORDS: Algebraic varieties, weight filtration, intersection homology.

1 Introduction

The mixed Hodge theory was developed by P. Deligne to study extraordinary properties of cohomology of complex algebraic varieties. One of the ingredients of mixed Hodge structure is the weight filtration. We will focus on the dual filtration in homology. We will find a relation between intersection homology defined by M. Goresky–R. MacPherson and the weight filtration. The main result of the paper says that if a variety is complete, then the pure part of homology $W^kH_k(X)$ is the image of intersection homology. This shows topological invariance of pure homology. An attempt to give a topological description or at least estimate the weight filtration was undertaken by C. McCrory ([10]). If X is a complete smooth divisor with normal crossings then the weight filtration coincides with the Zeeman filtration given by "codimension of cycles". In general there is an inclusion (the terms of the weight filtration are bigger). It is a result of McCrory for hypersurfaces and of F. Guillen ([8]) in general. For complete, normal and equidimensional varieties all we can deduce from Zeeman filtration for the pure term of the weight filtration is the following: the image of the Poincaré duality map is contained in $W^kH_k(X)$. This is exactly what follows from the weight principle. Our description of $W^kH_k(X)$ is complete and still purely topological. Other terms of the weight filtration do not have such description. They are not topologically invariant.

*Supported by KBN 2P03A 00218 grant. I thank Institute of Mathematics, Polish Academy of Science for hospitality.

In the course of the proof at some points we do not have to assume that the variety is complete. Therefore we organize the paper as follows. Let X be a complex algebraic variety. We will define certain subspaces of homology

$$IM_k(X) \subset H_k(X)$$

for each $k \in \mathbf{N}$, which we will call *image homology.* We assume that homology has coefficients in a field of characteristic zero. We will show that:

THEOREM 1.1 *The image homology satisfies the conditions:*

(A) *If X is smooth, then $IM_k(X) = H_k(X)$,*

(B) *If $f : X \to Y$ is an algebraic map then $f_* IM_k(X) \subset IM_k(Y)$,*

(C) *If $f : X \to Y$ is an algebraic map which is proper and surjective then $f_* IM_k(X) = IM_k(Y)$.*

By Hironaka resolution of singularities every algebraic variety is dominated by a smooth one. Therefore the conditions of Theorem 1.1 determine $IM_k(X)$. Of course it is not clear whether $IM_k(X)$ satisfying (A)-(C) exists. If X and Y are complete then the existence of such subspaces is a consequences of mixed Hodge theory, [5] §8.2.

THEOREM 1.2

(D) *If X is complete, then $IM_k(X)$ coincides with $W^k H_k(X)$, the pure weight subspace of homology.*

We prove that $IM_k(X)$ is the image of intersection homology:

THEOREM 1.3

(E) *If X is equidimensional, then*

$$IM_k(X) = im(\iota_X : IH_k(X) \to H_k(X)) .$$

We will explain the objects appearing in (D) and (E) below:

(d) $W^k H_k(X) = W_{-k} H_k(X)$ is the k-th term of the weight filtration in homology of X. It is the annihilator of $W_{k-1} H^k(X)$, the subspace of cohomology of the weight $\leq k-1$, defined in [5]. If X is complete then $W^k H_k(X)$ is the lowest possibly nonzero term. Then it is pure. In general the weight filtration is not a topologically invariant, see [12].

(e) $IH_k(X)$ is the intersection (co)homology group (with respect to the middle perversity) defined in [6, 7] and developed in [2]. It is a cohomology theory adapted to tackle singular varieties. It is equipped with canonical maps:

$$H_c^{2\dim(X)-k}(X) \xrightarrow{\iota_X^*} IH_c^{2\dim(X)-k}(X) = IH_k(X) \xrightarrow{\iota_X} H_k(X)$$

factorizing Poincaré duality map $[X] \cap -$. (The subscript c stands for compact supports.) Intersection (co)homology is a topological invariant of X. According to [2] or [11] one can construct a weight filtration in $IH_k(X)$. If X is complete, then $IH_k(X)$ is pure of weight k.

We deduce a striking property of image homology.

THEOREM 1.4 (TOPOLOGICAL INVARIANCE)

(F) *The image homology is a topological invariant of X. This means, that if $f : X \to Y$ is a homeomorphism which does not have to be an algebraic map then $f_* IM_k(X) = IM_k(Y)$.*

Theorem 1.4 follows from (E). As a corollary we obtain an unusual property of the pure term of the weight filtration:

COROLLARY 1.5 *If X is a complete algebraic variety, then $W^k H_k(X)$ is topologically invariant.*

A topological description of $W^k H_k(X)$ is the following

COROLLARY 1.6 *If X is a complete and equidimensional algebraic variety, then*

$$W^k H_k(X) = im(\iota_X : IH_k(X) \to H_k(X)) .$$

We also define parallel or dual variants of $IM_k(X)$:

- $IM_k^{BM}(X) \subset H_k^{BM}(X)$, a subspace of Borel-Moore homology satisfying the properties (A)-(F) (with (B) for proper maps),
- $KER^k(X) \subset H^k(X)$, *kernel cohomology*, a subspace of cohomology satisfying dual properties,
- $KER_c^k(X) \subset H_c^k(X)$, a subspace of cohomology with compact supports satisfying dual properties (with (B') for proper maps).

The dual properties are the following:

THEOREM 1.7 *The kernel cohomology $KER^k(X) \subset H^k(X)$ satisfies the conditions:*

(A') *If X is smooth, then $KER^k(X) = 0$,*

(B') *If $f : X \to Y$ is an algebraic map then $f^* KER^k(Y) \subset KER^k(X)$,*

(C') *If $f : X \to Y$ is an algebraic map which is proper and surjective then $(f^*)^{-1} KER^k(X) = KER^k(Y)$,*

(D') *If X is complete, then $KER^k(X) = W_{k-1} H^k(X)$, the subspace of cohomology of the weight $\leq k-1$,* (E') *If X is equidimensional, then*

$$KER^k(X) = ker(\iota_X^* : H^k(X) \to IH^k(X)),$$

(F') *The kernel cohomology is a topological invariant of X.*

Instead of KER groups one can consider the image of cohomology in intersection cohomology. We will focus on the case when X is equidimensional. The property (D') can be restated:

THEOREM 1.8

(D") *If X is complete and equidimensional then the image $im(\iota_X^* : H^k(X) \to IH^k(X))$ is isomorphic to the pure quotient $H^k(X)/W_{k-1}H^k(X)$.*

Combining (A') and (C') we obtain

COROLLARY 1.9 *If X is smooth, Y is equidimensional and the map $f : X \to Y$ is proper and surjective, then the image $f^*(H^*(Y))$ is isomorphic to the image ι_X^*.*

Corollary 1.9 (and an analogous one for homology) allows us to think about intersection cohomology as a substitute for the cohomology of a minimal resolution.

The main tool of the proofs is the Decomposition Theorem of [2] or [11]. We will rely only on one corollary:

COROLLARY 1.10 (FROM THE DECOMPOSITION THEOREM) *Let $f : X \to Y$ be a proper surjective map of algebraic varieties. Then IC_Y is a retract of Rf_*IC_X, i.e. there exist maps $i : IC_Y \to Rf_*IC_X$ (inclusion) and $r : Rf_*IC_X \to IC_Y$ (retraction) such that $r \circ i = Id_{IC_Y}$.*

Here IC_X is the intersection complex. It is an object of the derived category $D(X)$ of sheaves of vector spaces over X and X is assumed to have pure dimension n. The hypercohomology of IC_X is the intersection (co)homology:

$$H^k(X; IC_X) = IH^k(X) = IH^{BM}_{2n-k}(X),$$

$$H^k_c(X; IC_X) = IH^k_c(X) = IH_{2n-k}(X).$$

Due to methods of [2] we are forced to use complex coefficients (or $\overline{\mathbf{Q}}_\ell$), but once we prove the Theorems 1.1-1.4 for $\mathbf{C}$ they will automatically follow for arbitrary field of characteristic zero. We will also apply a result of [1] concerning (non canonical) functoriality of intersection (co)homology. As it is shown in [13] the result of [1] is a formal consequence of 1.10. On the other hand Corollary 1.10 can be deduced from [1]. We present this reasoning in the Appendix. This is the only non topological ingredient we use. The methods presented here are applied to study residues on singular hypersurfaces in [15].

2 Definition through (E)

The starting point of our definition is the property (E). We cannot say that $IM_k(X)$ is just the image of $IH_k(X)$ since intersection homology is defined only for equidimensional varieties. Therefore we decompose X into irreducible components $X = \bigcup_{i\in I} X_i$.

DEFINITION 2.1 *Let $\iota_i : IH_k(X_i) \to H_k(X)$ be the composition of the natural transformation ι_{X_i} with the map induced by the inclusion of the component. The image homology of X is defined by*

$$IM_k(X) = \sum_{i\in I} im(\iota_i) \subset H_k(X)\,.$$

If X is equidimensional we do not have to decompose X into components since

$$IH_k(X) = IH_k(\widehat{X}) = \bigoplus_{i\in I} IH_k(X_i)\,,$$

where $\widehat{X}$ is the normalization of X, which is also the normalization of $\bigsqcup_{i\in I} X_i$. This proves the property (E).

We can give a direct description of image homology in terms of cycles. Our description is based on the elementary definition of intersection homology introduced in [6]. We fix a stratification $\{S_\alpha\}_{\alpha\in J_i}$ of each X_i. We assume that the stratification is locally topologically trivial. A homology class of $H_k(X)$ belongs to $IM_k(X)$ if it is represented by a cycle ξ which

- can be decomposed into a sum of cycles $\xi = \sum_{i\in I} \xi_i$, with $|\xi_i| \subset X_i$.
- the component ξ_i is allowable in X_i, i.e.

  $$\dim_{\mathbf{R}}(|\xi_i| \cap S_\alpha) \le k - c - 1$$

  if S_α is a singular stratum of X_i with complex codimension $c = \dim(S_\alpha) - \dim(X_i)$.

3 Proof of (F)

By [7] or [9] intersection homology is a topological invariant. It is enough to show that the decomposition into irreducible components is topologically invariant. Indeed, let X_{reg} be the set of points of X at which X is a topological manifold (the dimension may vary from a point to a point). The set X_{reg} decomposes into a set of components $X_{i,\text{reg}}$. The irreducible components of X are the closures of $X_{i,\text{reg}}$. □

4 Proof of (A)

If X is smooth, then $\iota_X : IH_k(X) \to H_k(X)$ is an isomorphism. □

Remark 4.1 It is enough to assume that X is a rational homology manifold to have $IM_k(X) = H_k(X)$. Also some information about local homology allows to deduce an isomorphism in certain degrees. For example if X has isolated singularities, then $IM_k(X) = H_k(X)$ for $k > \max_i(\dim(X_i))$.

5 Proof of (B)

At this point for the first time we have to use highly nontrivial results concerning intersection homology. Let $f : X \to Y$ be an algebraic map. Let us recall the main theorem of [1].

THEOREM 5.1 *There is a map β making the following diagram commute:*

$$\begin{array}{ccc} IH_k(X) & \xrightarrow{\iota_X} & H_k(X) \\ {\scriptstyle\beta}\downarrow & & \downarrow{\scriptstyle f_*} \\ IH_k(Y) & \xrightarrow{\iota_Y} & H_k(Y) \end{array} .$$

The choice of β is not canonical, but its existence is enough to deduce (B). □

In some cases we can prove more:

PROPOSITION 5.2 *If X is complete, then f_* strictly preserves image homology:*

$$im(f_*) \cap IM_k(Y) = f_*(IM_k(X)) .$$

We will deduce 5.2 from mixed Hodge theory of [5] provided we know (C) and (D).

Proof. Let $\mu : \widetilde{Y} \to Y$ be a resolution of singularities. By (C) $IM_k(Y)$ is the image of μ_*. Therefore $IM_k(Y) \subset W^k H_k(Y)$. The map f_* strictly preserves weights

$$im(f_*) \cap W^k H_k(Y) = f_*(W^k H_k(X)) .$$

By (D) we have $W^k H_k(X) = IM_k(X)$. We obtain the inclusion

$$im(f_*) \cap IM_k(Y) \subset f_*(IM_k(X)) .$$

The converse inclusion follows from (B). □

6 Proof of (C)

We assume that $f : X \to Y$ is proper and surjective. It is enough to prove (C) for irreducible X and Y. We will show that the map $\iota_Y : IH_k(Y) \to H_k(Y)$ factors through $IH_k(X)$. Our argument is dual to the one of [13]. We will construct a map α which will fit to the commutative diagram

$$\begin{array}{ccc} IH_k(X) & \xrightarrow{\iota_X} & H_k(X) \\ \alpha\uparrow & & \downarrow f_* \\ IH_k(Y) & \xrightarrow{\iota_Y} & H_k(Y) \end{array} .$$

It will come from a map $\overline{\alpha}$ at the level of derived category of sheaves:

$$\begin{array}{ccc} Rf_*IC_X[2n] & \xrightarrow{Rf_*(\iota_X)} & Rf_*\mathbb{D}_X \\ \overline{\alpha}\uparrow & & \downarrow f_* \\ IC_Y[2m] & \xrightarrow{\iota_Y} & \mathbb{D}_Y \end{array} .$$

Here $n = \dim(X)$, $m = \dim(Y)$ and $\mathbb{D}_X$ (resp. $\mathbb{D}_Y$) denotes the dualizing sheaf. It is equal to $\mathbf{C}_X[2n]$ (resp. $\mathbf{C}_Y[2m]$) at smooth points. By the Corollary from the Decomposition Theorem 1.10 there is a retraction $r : Rf_*IC_X \to IC_Y$. Let

$$\overline{\alpha} : IC_Y[2m] = DIC_Y \xrightarrow{Dr} DRf_*IC_X = Rf_*DIC_X = Rf_*IC_X[2n]$$

be the Verdier dual of the retraction. We obtain two maps: ι_Y and the composition $f_* \circ Rf_*(\iota_X) \circ \overline{\alpha} : IC_Y[2m] \to \mathbb{D}_Y$. We can rescale $\overline{\alpha}$ and assume that these two maps coincide on an open set. We will show that they coincide on the whole X. The conclusion will follow.

PROPOSITION 6.1 *Suppose Y is irreducible. Then every two maps in the derived category $\iota_1, \iota_2 : IC_Y \to \mathbb{D}_Y$ which coincide on an open set are equal.*

Proof. It is more convenient to prove the dual statement: *Every two maps $D\iota_1, D\iota_2 : \mathbf{C}_Y \to IC_Y$ which coincide on an open set are equal.* Indeed

$$\mathrm{Hom}_{D(Y)}(\mathbf{C}_Y, IC_Y) = R^0Hom(\mathbf{C}_Y, IC_Y) = H^0(Y; IC_Y) = IH^0(Y) \simeq \mathbf{C}$$

and the same for U. The restriction map $IH^0(Y) \to IH^0(U)$ is an isomorphism. □

Remark 6.2 Another possible proof is to show that β in 5.1 may be chosen to be surjective.

Remark 6.3 Since the factorization of ι_Y was constructed on the sheaf level we obtain the statement for arbitrary supports of the homology:

$$f_*IM_k^{f^{-1}(\phi)}(X) = IM_k^{\phi}(Y) \subset H_k^{\phi}(Y),$$

where ϕ is a family of supports in Y. In particular

$$f_* IM_k^{BM}(X) = IM_k^{BM}(Y) \subset H_k^{BM}(Y)\,,$$

provided that the map f is proper.

Remark 6.4 Every algebraic subvariety $A \subset Y$ of dimension k defines a class in $IM_{2k}^{BM}(Y)$. It is easy to show a class in $IM_{2k}^{BM}(X)$ which is mapped to $[A]$. To construct it we choose an open and dense subset $U \subset A$ and a subvariety $B \subset f^{-1}U$ such that $f_{|B} : B \to U$ is finite, say of degree d. Let C be the closure of B in X. Then the class $\frac{1}{d}[C] \in IM_{2k}^{BM}(X)$ is the desired lift of $[A]$.

Remark 6.5 The property (C) does not hold for the maps which are not algebraic. Example: Let N be the rational node, i.e. $\mathbf{P}^1$ with two points identified. It is homeomorphic with the famous pinched torus, i.e. $S^1 \times S^1$ with $S^1 \times pt$ shrunk. The quotient map $S^1 \times S^1 \to N$ is surjective on $H_1(N) \simeq \mathbf{C}$, but $IM_1(N) = H_1(\mathbf{P}^1) = 0$.

Let us note that the property (C) does not hold in the analytic category, even in the smooth case.

Example 6.6 Let $X = (\mathbf{C}^2 \setminus \{0\})/\mathbf{Z}$, where $k \in \mathbf{Z}$ acts on $\mathbf{C}^2$ via the multiplication by 2^k. The tautological bundle factors through $f : X \to \mathbf{P}^2 = Y$. The map f is a complex analytic locally trivial fibration with fibers $\mathbf{C}^*/\mathbf{Z}$ which is an elliptic curve (a compact torus). Topologically $f : X \simeq S^3 \times S^1 \to S^2 \simeq Y$ is the projection on the first factor composed with the Hopf fibration. The induced map is not surjective on $H_2(Y)$.

7 Proof of (D)

We recall that the weight filtration in the homology of a complete variety is such, that $W^{k+1}H_k(X) = 0$ and $W^k H_k(X)$ is pure of weight k. Our proof is based on the following description of $W^k H_k(X)$ (see [10] p.218): Assume that X is contained in a smooth variety M. Let $\mu : \widetilde{M} \to M$ be a resolution of (M, X) in the sense that $\mu^{-1}X$ is a smooth divisor with normal crossings. Let $\overline{X}$ be the disjoint union of the components of $\mu^{-1}X$. Then we have an equality

$$W^k H_k(X) = \mu_*(W^k H_k(\mu^{-1}X)) = \nu_*(H_k(\overline{X}))\,,$$

where $\nu : \overline{X} \to X$ is the obvious map. On the other hand by (A) and (C)

$$\nu_*(H_k(\overline{X})) = IM_k(X)\,.$$

□

Remark 7.1 By the dual of [5], 8.2.5 we can replace $\nu : \overline{X} \to X$ by any surjective map from a complete and smooth variety.

Remark 7.2 In general, if X is possibly not complete then

$$IM_k(X) \subset W^k H_k(X)\,.$$

The equality does not have to hold. Example: Let $X = \mathbf{C}^* \times N$, where N is the rational node (see 6.5). Then $W^2H_2(X) = H_2(X) \simeq \mathbf{C}^2$, but $IM_2(X) \simeq H_2(\mathbf{C}^* \times \mathbf{P}^1) \simeq \mathbf{C}$.

8 Borel-Moore homology

Proof of the properties (A), (D), (E) and (F) are the same. Concerning (B) and (C), as we have remarked in 6.3, we can use any kind of supports, since the maps are constructed on the level of sheaves.

9 Dual *KER* groups

Intersection cohomology is a module over $H^*(X)$. Since the natural transformation ι_X preserves the module structure we have:

PROPOSITION 9.1 *The kernel* $ker(\iota_X) = KER^*(X)$ *is an ideal in the cohomology ring* $H^*(X)$. *Moreover* $KER^0(X) = 0$.

Due to nondegeneracy of the pairing in intersection (co)homology we can describe kernel cohomology as annihilators

$$KER^k(X) = Ann(IM_k(X))\,, \qquad KER^k_c(X) = Ann(IM^{BM}_k(X))\,.$$

Properties of (A')-(F') can be obtained by duality. A short way to prove (C') is to show that β in the dual of the diagram 5.1 may be chosen to be injective. The inclusion of 1.10 is a good choice. We will comment on (D'): We recall that the weight filtration in the cohomology of a complete variety is such, that $W_kH^k(X) = H^k(X)$ and the quotient $W_kH^k(X)/W_{k-1}H^k(X)$ is pure. Combining (D') and (E') we have:

THEOREM 9.2 *If* X *is complete and equidimensional, then*

$$W_{k-1}H^k(X) = ker(\iota^*_X : H^k(X) \to IH^k(X))$$

This gives a positive answer to a question stated in [4].

10 Differential

By the property (B) the image homology is functorial. (We recall that it is not possible to chose maps of intersection homology to make it functor, see [13].) Moreover the map $\oplus\iota_i : \bigoplus_{i\in I} IH_k(X_i) \to H_k(X)$ comes from a map of sheaves $\oplus\iota_i : \bigoplus_{i\in I} IC_{X_i} \to \mathbb{D}_X$. Therefore $\partial(IM_k(X)) \subset IM_{k-1}(A\cap B)$ in the Mayer-Vietoris sequence associated to the open covering $X = A\cup B$. Of course the sequence of IM groups does not have to be exact. Example: Let X the sum of two copies of $\mathbf{P}^1$ glued along two pairs points. Denote by a and b be the points of the identification. The Mayer-Vietoris sequence for $A = X\setminus\{b\}$, $B = X\setminus\{a\}$ is not exact at $IM_0(A\cap B)$.

On the other hand one can consider the case when A and B are closed subvarieties. Then the differential for intersection homology vanishes.

11 Appendix

The most frequently applied corollary from the Decomposition Theorem is 1.10. Its proof does not demand all the machinery of [2]. The Corollary 1.10 can be deduced from functoriality of intersection cohomology, which in turn relies only on a certain vanishing statement called *Local Hard Lefschetz* in [14]. We want to show how to construct inclusion and retraction in 1.10 using the induced maps constructed in [1].

We assume that X and Y are irreducible, $\dim(X) = n$ and $\dim(Y) = m$. The map $f : X \to Y$ is proper and surjective.

INCLUSION: $IC_Y \xrightarrow{i} Rf_*IC_X$ is a map realizing $f^* : IH^*(Y) \to IH^*(X)$. Such a map exists by [1]

RETRACTION: $Rf_*IC_X \xrightarrow{r} IC_Y$. Let Z be a subvariety of X such, that $f_{|Z}$ is surjective and $\dim(Z) = \dim(Y)$. Such a construction has already been applied in Remark 6.4 and [1] p. 173. We consider the sheaf IC_Z as a sheaf on X supported by Z. As in §6 we construct a map $\alpha : IC_Y \to Rf_*IC_Z$. We obtain a map of Verdier dual sheaves

$$Rf_*IC_Z[2m] = Rf_*DIC_Z = DRf_*IC_Z \xrightarrow{D\alpha} DIC_Y = IC_Y[2m]\,.$$

Again, there exists a map $\beta : IC_X \to IC_Z$ which agrees with the restriction of the constant sheaf $\mathbf{C}_X \to \mathbf{C}_Z$. We compose $Rf_*\beta$ with $D\alpha[-2n]$ and obtain a map $r' : Rf_*IC_X \to IC_Y$. Generically the composition $r'\circ i$ is the multiplication by the degree of $f_{|Z}$. We define $r = \deg(f_{|Z})^{-1}r'$. Now, $r\circ i : IC_Y \to IC_Y$ is generically identity. By [3], V.9.2 p.144 each morphism of intersection complexes is determined by its restriction to an open set, therefore $r\circ i = Id_{IC_Y}$. □

Remark 11.1 The existence of the induced maps constructed in [1] follows from local vanishing (Local Hard Lefschetz [14]). Therefore these maps exist with rational coefficients. Our inclusion and retraction are constructed for rationals as well.

Remark 11.2 The assumption that we stay in the algebraic category is essential. In the Example 6.6 there is a natural map $\mathbf{C}_Y = IC_Y \to Rf_* IC_X = Rf_* \mathbf{C}_X$, but it is not split injective. The construction of a retraction breaks down since one cannot find a subvariety $Z \subset X$ with the desired properties.

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