PLÜCKER FORMULAS AND MOTIVIC CHERN CLASSES OF COINCIDENT ROOT LOCI

Joint work with András Juhász.

Suppose that $f \in \mathrm{Pol}^d(\mathbb{C}^3)$, i.e. f is a degree d homogeneous polynomial in 3 variables. Then f defines a projective plane curve $Z_f \subset \mathbb{P}^2$. Assuming that Z_f is non-singular a famous formula of Plücker tells us that Z_f has $3d(d-2)$ flexes. This can be reproved using equivariant cohomology classes of coincident root loci (which were calculated 15 years ago by Fehér-Némethi-Rimányi and Kőműves): if $g \in \mathrm{Pol}^d(\mathbb{C}^2)$, then g is a product of d linear factors, and $\mathrm{Pol}^d(\mathbb{C}^2)$ is the disjoint union of strata Y_λ for partitions of d, according to the multiplicities. For example the equivariant cohomology class of $Y_{3,1,\dots,1}$ is

$$d(d-2)\big((d-1)c_1^2 - (d-4)c_2\big).$$

A simple geometric argument gives that the number of flexes can be obtained by substituting $c_1^2 = c_2 = 1$, obtaining the Plücker formula above.

We can study similar problems in higher dimensions connected to tangent lines to degree d hypersurfaces, and there is a family of generalized Plücker formulas which contain the same information as the equivariant cohomology classes of coincident root loci.

The last Plücker formula calculates the Euler characteristics of the dual curve (which is the set of tangent lines) of Z_f. In modern terms this can be calculated from the equivariant Chern-Schwartz-MacPherson class $\mathrm{c}^{\mathrm{SM}}(\bar{Y}_{2,1,\dots,1})$. The CSM class is a refinement of the cohomology class. The CSM classes of coincident root loci were calculated recently by Balázs Kőműves. It turns out that there is a wider class of Plücker formulas concerning the Euler characteristics of certain tangent line varieties which contain the same information as the CSM classes of coincident root loci.

The next natural step is to obtain information on the motivic χ_y-genus of these tangent line varieties. This can be done by calculating the motivic Chern classes of coincident root loci. We start to discover that there is an elaborate network of dependencies among these numbers governed by the geometry.

The lecture is meant to be introductory, I try to explain how these modern tools help to solve classical and less classical enumerative problems. A basic level of understanding of Chern classes and projective varieties should be enough to enjoy the talk.