**Local structure of finite groups and of their classifying spaces**

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Let $p$ be a prime. By the "$p$-local structure" of a finite group $G$ is meant a Sylow $p$-subgroup $S \in \mathrm{Syl}_p(G)$ and the $G$-conjugacy relations among the subgroups of $S$. More precisely, two finite groups $G$ and $H$ are said to have the same $p$-local structure if there is an isomorphism $S \xrightarrow{\cong} T$, for some $S \in \mathrm{Syl}_p(G)$ and some $T \in \mathrm{Syl}_p(H)$, that preserves all $G$- and $H$-conjugacy relations among subgroups of $S$ and of $T$.

A "classifying space" of a finite group $G$ is an Eilenberg-MacLane space of type $K(G, 1)$; i.e., a topological space $BG$ whose fundamental group is isomorphic to $G$ and whose universal covering space is contractible. Two classifying spaces $BG$ and $BH$ are said to be "$p$-locally equivalent" if there is a third space $X$, and maps $f\colon BG \longrightarrow X$ and $f'\colon BH \longrightarrow X$, such that $f$ and $f'$ both induce isomorphisms in homology with coefficients in $\mathbb{Z}/p$.

A conjecture by Martino and Priddy (1996), now a theorem, says that for each pair of finite groups $G$ and $H$ and each prime $p$, $G$ and $H$ have the same $p$-local structure if and only if their classifying spaces are $p$-locally equivalent. The first proofs of this theorem, by me (2006) and by Chermak (2013), depended on the classification of finite simple groups. But in more recent work, Glauberman and Lynd (2016) have shown how to prove it directly, independent of that classification.

In the talk, I want to first describe in more detail the background of this theorem, and also talk very briefly about some of the ideas behind its proof. I'll then describe a related result involving automorphisms of groups and of their classifying spaces. If there's time left, I'll also give a few examples and applications of both of these, especially in the case where $G$ and $H$ are finite simple groups of Lie type.