Algorithmic Trends
Homework 5

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The homework is due on 14/05/2014.

Problem 1

Consider the following auction problem that is based on the knapsack problem. We are given n players. Every player has a private valuation v_i and publicly known size c_i. Moreover, the size C of the knapsacks in known publicly as well. The feasible allocation is given by a subset S of players, such that $\sum_{i \in S} c_i \leq C$. We assume that $c_i \leq C$ for all $1 \leq i \leq n$. The problem of computing the best feasible allocation corresponds exactly to the knapsack problem. The value of the allocation S is given by $\sum_{i \in S} v_i$.

- Consider the following algorithm for this problem. First, sort the players according to the decreasing valuation and greedily pack the knapsack according to this order. Let S_1 be the obtained allocation. Next, sort the players in a nondecreasing order according to the ratio v_i/c_i, and greedily pack the knapsack according to this order. Let S_2 be the allocation obtained this way. The algorithm returns the better one of the two allocations S_1 and S_2. Prove that this algorithm is 1/2-approximate?
- Show how to construct and incentive compatible auction using this algorithm? The auction should be incentive compatible with respect to the valuations only.

Problem 2

We are going to sell one item to n players in the "gender equal" way. In this set of n players n_B are boys and n_G are girls. ($n_B, n_G > 0$ and $n_B + n_G = n$). Consider the following auction:

- every player submits his/her bid,
- let b_B be the boy that submitted the highest bid,

- let b_G be the girl that submitted the highest bid,
- the seller tosses a coin and:
  - with probability 1/2 the boy b_B wins and pays the bid submitted by girl b_G,
  - with probability 1/2 the girl b_G wins and pays the bid submitted by boy b_B.

Prove that the auction is incentive compatible or show an example where it is not incentive compatible?

Problem 3

In the atomic splittable selfish routing game every player controls r_i units of flow, which can be divided and routed in an arbitrary way on paths from s_i to t_i. Given such game we can obtain a new game by replacing each player by two players that want to route $r_i/2$ units of flow from s_i to t_i. This operation does not change the cost of an optimal flow. Prove that this splitting operation can reduce the price of anarchy?