Algorithmic Trends
Homework 4

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The homework is due on 14/05/2014.

Problem 1

The separator theorem gives us a division of an n-vertex planar graph into three parts A, B and C, which satisfy the following conditions:

- there is no edge in G between A and B,
- A and B contain no more then $\frac{2}{3}n$ fraction of vertices,
- C contains no more then $2\sqrt{2n}$ vertices.

Consider the division of a graph G into two parts induced by $A \cup C$ and $B \cup C$. Apply this division in a recursive way until you obtain parts containing $O(1)$ vertices. Analyze:

- the number of recursive levels of this procedure?
- give an upper bound on the total size of obtained graphs on each level of the recursion?

Problem 2

Let $G = (V, E)$ be a graph with edge weights given by a function $w : E \to \mathcal{R}^+$. In the *maximum cut* problem we want to find a subset $S \subseteq V$, which maximizes the weight of edges going between S and $V - S$, i.e., $w(S, V - S)$. In the *minimum odd-length cycle cover* problem we want to find a set of edges D having the minimum weight such that after removing D from G no odd length cycle in G is left. Prove that in a planar graph the set of edges S is the maximum cut if and only if it is the compliment of the minimum odd-length cycle cover.

Problem 3

Let G be a simple, connected planar graph on n vertices ($n \geq 3$), that has m edges and let g be the length of the shortest cycle in G. Prove that:

$$m \leq \frac{g(n-2)}{g-2}.$$