Algorithmic Trends

Homework 3

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April 8, 2014
The homework is due on April 23, 2014

Problem 1. Assume that we are given a treen T with k vertices, as well as an undirected graph G. We want to check whether T is a subgraph of G.

- Show how to solve this problem in $c^k n^{\mathcal{O}(1)}$ time.

- Show how to solve this problem in $2^k n^{\mathcal{O}(1)}$ time.

Problem 2. Show that one can check whether G contains a clique of size k in time $\mathcal{O}(n^{\delta k})$ for some $\delta < 1$ (hint: use fast matrix multiplication).

Problem 3. Consider the following problem. The input consists of an alphabet Σ, an integer k and k^2 subsets $A_{i,j} \subseteq \Sigma \times \Sigma$ for $1 \leqslant i, j \leqslant k$. The goal is to check whether there are functions $x : \{1, \ldots, k\} \to \Sigma$ and $y : \{1, \ldots, k\} \to \Sigma$ such that for each $1 \leqslant i, j \leqslant k$ we have $(x_i, y_j) \in A_{i,j}$.

Show that this problem is $W[1]$-hard when parameterized by k.