Algorithmic Trends
Homework 1

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The homework is due on 26/03/2014.

Problem 1

Let $G = (V, E)$ be a directed graph with two fixed vertices $s$ and $t$. Propose an algorithm that computes the maximum number of vertex disjoint paths from $s$ to $t$. The vertices $s$ and $t$ can be shared on these paths. Hint: Reduce the problem to the maximum matching problem.

Problem 2

Let $G = (V, E)$ be a graph. Assume that you are given an algorithm $A$ that checks for every pair of nodes $x, y \in V$ whether there exists a perfect matching in $G - x - y$. Show how to use this algorithm to multiply two boolean matrices of size $|V| \times |V|$ in asymptotically the same time as the running time of $A$.

Problem 3

Let $G$ be a graph. Prove that if a vertex set $S \subseteq V(G)$ is covered by some matching, then there exists a maximum size matching that covers $S$. 