

Mathematical analysis 2, WNE, 2018/2019  
meeting 27.

6 June 2019

1. a) Calculate integral

$$\int \frac{\cos x}{1 + \sin^2 x} dx.$$

- b) Calculate improper integral

$$\int_0^\infty x e^{-x} dx.$$

2. Let  $f: \mathbb{R}^2 \rightarrow \mathbb{R}$  be defined by the formula

$$f(x, y) = x^3 + y^3 + 6xy + 3.$$

Find all points  $(x, y) \in \mathbb{R}^2$  such that  $\nabla f(x, y) = (0, 0)$  and determine which are local minima, local maxima or saddle points.

3. Let  $F: \mathbb{R}^3 \rightarrow \mathbb{R}$  be defined by the formula

$$F(x, y, z) = x \exp(z) - 2 + \cos(yz).$$

- a) Prove that there exists a neighbourhood of  $(x, y) = (1, 1)$  on which it is possible to define a function  $z = z(x, y)$  of  $C^1$  class such that  $z(1, 1) = 0$  and  $F(x, y, z(x, y)) = 0$ . Calculate  $\frac{\partial z}{\partial x}(1, 1)$  and  $\frac{\partial z}{\partial y}(1, 1)$ .  
b) Find an equation of a plane tangent to

$$M = \{(x, y, z) \in \mathbb{R}^3: F(x, y, z) = 0\}$$

at  $(1, 1, 0)$ .

4. Find the maximal and minimal value of  $f(x, y, z) = 2x - 4y + 2z$  under conditions

$$g_1(x, y, z) = x^2 + y^2 + z^2 - 1 = 0,$$

$$g_2(x, y, z) = x + y + z = 0.$$

5. Calculate double integrals:

- a)

$$\iint_{x^2 + y^2 \leq 1} \sin(x^2 + y^2) dx dy,$$

- b)

$$\int_0^1 \int_y^1 e^{-x^2} dx dy.$$