Mathematical analysis 2, WNE, 2018/2019
meeting 25.

28 May 2019

1. Is there a function of C^2 class, $f\colon \mathbb{R}^2 \to \mathbb{R}$, such that
$$\frac{\partial f}{\partial x} = x \sin y,$$
$$\frac{\partial f}{\partial y} = y \cos x?$$

2. Let $f(x,y) = x + y^2$. Find the maximum of f on the set
$$A = \left\{(x,y) \in \mathbb{R}^2 :\ 3x^2 + 2y^2 + 8x \leqslant 1\right\}.$$

3. Let $K = \{(x,y) \in \mathbb{R}^2 : 0 \leqslant x \leqslant 1,\ 0 \leqslant y \leqslant 2\}$ and let $f(x,y) = x^2 + y - xy^2$. Find the maximum and the minimum of f on the set K.

4. Find and classify the critical points of the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by
$$f(x,y) = x^3 + y^3 + 3xy + 3.$$

5. Let $f(r,\theta) = \big(r^2\cos(2\theta), r^2\sin(2\theta)\big)$ where $(r,\theta) \in (0,1) \times (0,2\pi)$. Sketch the image of f.

6. Let f be the function from the previous problem.
   a) Is f a local diffeomorphism of the set $(0,1) \times (0,2\pi)$?
   b) Is f a diffeomorphism of the set $(0,1) \times (0,2\pi)$?

7. Let $z(x,y)$ be determined by the equation $\sin(xz) = yz$ and $z(1,0) = 0$. Calculate $\frac{\partial z}{\partial x}(1,0)$ and $\frac{\partial z}{\partial y}(1,0)$.

8. Find the equation of the plane tangent to the surface
$$S = \{(x,y,z) \in \mathbb{R}^3 :\ \sin(xz) = yz\}.$$
at the point $(1,0,0)$.

9. Is the following subset of $\mathbb{R}^3$ a manifold
$$M = \{(x,y,z) \in \mathbb{R}^3 :\ x^2 + y^2 + z^2 = 1,\ \ z = x^2 + y^2\}.$$

10. Decide whether the quadratic form given by the matrix $\begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is: positive definite, positive semi-definite, negative definite, negative semidefinite, or indefinite.