Mathematical analysis 2, WNE, 2018/2019
meeting 23. – homework solutions

21 May 2019

Group 8:00

Among the points which belong to the intersection of the plane $x + y + z = 12$ and the paraboloid $z = x^2 + y^2$ find those closest and farthest from the origin.

We study $f(x, y, z) = x^2+y^2+z^2$ for $F(x, y, z) = (x+y+z-12, x^2+y^2-z) = (0, 0)$. Thus, $f' = (2x, 2y, 2z)$, $F_1' = (1, 1, 1)$ and $F_2' = (2x, 2y, -1)$. Hence,

$$\begin{cases} 2x = \lambda_1 + 2\lambda_2 x \\ 2y = \lambda_1 + 2\lambda_2 y \\ 2z = \lambda_1 - \lambda_2 \\ x + y + z = 12 \\ z = x^2 + y^2 \end{cases}$$

If $\lambda_2 = 1$, to $\lambda_1 = 0$, then $z = -1/2$, which is impossible, because by the last equation, $z \geqslant 0$. Therefore,

$$x = \frac{\lambda 1}{2 - 2\lambda_2} = y$$

Thus, $z = 12 - 2x$ and $12 - 2x = 2x^2$, so $x^2 + x - 6 = 0$ and $x = y = 2$ or $x = y = -3$. Then $z = 8$ or $z = 18$ respectively, so the points are $(2, 2, 8)$ i $(-3, -3, 18)$. The values of f are 72 and 342 respectively, so $(2, 2, 8)$ is the nearest and $(-3, -3, 18)$ is the furthest.

Grupa 9:45

Among the points which belong to the intersection of the plane $x + y + z = 12$ and the paraboloid $x = y^2 + z^2$ find those closest and farthest from the origin.

We study $f(x, y, z) = x^2+y^2+z^2$ for $F(x, y, z) = (x+y+z-12, -x+y^2+z^2) = (0, 0)$. So $f' = (2x, 2y, 2z)$, $F_1' = (1, 1, 1)$ and $F_2' = (-1, 2y, 2z)$. Thus,

$$\begin{cases} 2x = \lambda_1 - \lambda_2 \\ 2y = \lambda_1 + 2\lambda_2 y \\ 2z = \lambda_1 + 2\lambda_2 z \\ x + y + z = 12 \\ x = y^2 + z^2 \end{cases}$$

If $\lambda_2 = 1$, to $\lambda_1 = 0$, then $x = -1/2$, which is impossible, because by the last equation, $x \geqslant 0$. Hence,

$$y = \frac{\lambda 1}{2 - 2\lambda_2} = z$$

Therefore, $x = 12 - 2y$ and $12 - 2y = 2y^2$, so $y^2 + y - 6 = 0$ i $y = z = 2$ or $y = z = -3$. Then $x = 8$ or $x = 18$ respectively, so the points are $(8, 2, 2)$ and $(18, -3, -3)$. The values of f are 72 and 342 respectively, so $(8, 2, 2)$ is the nearest and $(18, -3, -3)$ is the furthest.