

Mathematical analysis 2, WNE, 2018/2019 meeting 22.

16 May 2019

Problems

1. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function of C^1 class such that for some $0 < k < 1$ and all $x \in \mathbb{R}$, $|f'(x)| \leq k$. Prove that $y = x + f(x)$ is a diffeomorphism.

2. Let

$$E = \{(x, y) \in \mathbb{R}^2: x^2 - 2xy + 2y^2 = 1\}.$$

Use the Lagrange multipliers method to find points of E which are closest to and farthest from the origin of the coordinate system.

3. Use the Lagrange multipliers method to find all those points on the ellipse $x^2 + 2y^2 = 1$, which are nearest to and furthest from the line $x + y = 2$.

4. Find supremum and infimum of $f(x, y, z) = x^2 - yz$ on the sphere $x^2 + y^2 + z^2 = 1$.

5. Find the minimal value of $f(x, y, z) = x + y + z$ on the sphere $x^2 + y^2 + z^2 = a^2$.

6. Prove the following inequality between the arithmetic and square mean, i.e.

$$\frac{x + y + z}{3} \leq \sqrt{\frac{x^2 + y^2 + z^2}{3}},$$

for $x, y, z \geq 0$.

Homework

Group 8:00

Find the maximal and minimal values of $f(x, y) = x^2 - y^2$ on the set $\{(x, y) \in \mathbb{R}^2: x^2 + y^2 = 4\}$.

Group 9:45

Find the maximal and minimal values of $f(x, y) = 4x^2 + 9y^2$ on the set $\{(x, y) \in \mathbb{R}^2: x^2 + y^2 = 1\}$.