Mathematical analysis 2, WNE, 2018/2019
meeting 22. – homework solutions

16 May 2019

Group 8:00

Find the maximal and minimal values of $f(x, y) = x^2 - y^2$ on the set $\{(x, y) \in \mathbb{R}^2 \colon x^2 + y^2 = 4\}$.

$f' = [2x, -2y]$, $F(x, y) = x^2 + y^2 - 4 = 0$, $F'(x, y) = [2x, 2y]$. Thus we are looking for points such that $2x = \lambda 2x$, $-2y = \lambda 2y$. If $x = 0$, then $y = \pm 2$ (and $\lambda = -1$), but if $x \neq 0$, then $\lambda = 1$, so $y = 0$ and $x = \pm 2$. So we get points $(0, 2)$, $(0, -2)$, $(2, 0)$, $(-2, 0)$ and the values are $-4, -4, 4$ and 4 respectively. So -4 is the minimal value and 4 is the maximal.

Group 9:45

Find the maximal and minimal values of $f(x, y) = 4x^2 + 9y^2$ on the set $\{(x, y) \in \mathbb{R}^2 \colon x^2 + y^2 = 1\}$.

$f' = [4x, 9y]$, $F(x, y) = x^2 + y^2 - 1 = 0$, $F'(x, y) = [2x, 2y]$. Thus we are looking for points such that $4x = \lambda 2x$, $9y = \lambda 2y$. If $x = 0$, then $y = \pm 1$ (and $\lambda = 9/2$), but if $x \neq 0$, then $\lambda = 2$, so $y = 0$ and $x = \pm 1$. So we get points $(0, 1)$, $(0, -1)$, $(1, 0)$, $(-1, 0)$ and the values are $9, 9, 4$ and 4 respectively. So 4 is the minimal value and 9 is the maximal.