Mathematical analysis 2, WNE, 2018/2019
meeting 21. – homework solutions

14 May 2019

Group 8:00

Using the implicit function theorem check whether there exists a function $H\colon \mathbb{R}^2 \to \mathbb{R}^2$ such that if (x, y, z, t) is close enough to $(0, 1, 0, \pi/2, 1)$, then

$$\begin{cases} \sin xy + \cos zt = 0 \\ x - y + z - t - \pi/2 = 0 \end{cases}$$

if and only if $H(x, y) = (z, t)$. If such a function exists, find $H'(0, 1)$.

We have $F(x, y, z, t) = (\sin xy + \cos zt, x - y + z - t - \pi/2)$ and $F(x, y, z, t) = (0, 0, 0, 0)$, and

$$F'(x, y, z, t) = \begin{bmatrix} y\cos xy & -x\cos xy & -t\sin zt & -z\sin zt \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

so

$$F'(0, 1, \pi/2, 1) = \begin{bmatrix} 0 & 1 & -1 & -\pi/2 \\ 1 & -1 & 1 & -1 \end{bmatrix},$$

we check that

$$\det \begin{bmatrix} -1 & -\pi/2 \\ 1 & -1 \end{bmatrix} = 1 + \pi/2 \neq 0.$$

Thus H exists and

$$H'(0, 1) = -\begin{bmatrix} -1 & -\pi/2 \\ 1 & -1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} =$$

$$\frac{1}{1 + \pi/2}\begin{bmatrix} -1 & -\pi/2 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} = \frac{1}{1 + \pi/2} \cdot \begin{bmatrix} -\pi/2 & -1 + \pi/2 \\ -1 & 2 \end{bmatrix}.$$

Group 9:45

Using the implicit function theorem check whether there exists a function $H\colon \mathbb{R}^2 \to \mathbb{R}^2$ such that if (x, y, z, t) is close enough to $(\pi/2, 1, 1, 0)$ to

$$\begin{cases} \cos xy + \sin zt = 0 \\ x - y + z - t - \pi/2 = 0 \end{cases}$$

if and only if $H(x, y) = (z, t)$. If such a function exists, find $H'(\pi/2, 1)$.

We have $F(x, y, z, t) = (\cos xy + \sin zt, x - y + z - t - \pi/2)$ i $F(x, y, z, t) = (0, 0, 0, 0)$, and

$$F'(x, y, z, t) = \begin{bmatrix} -y\sin xy & -x\sin xy & t\cos zt & z\cos zt \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

so

$$F'(\pi/2, 1, 0, 1) = \begin{bmatrix} -1 & -\pi/2 & 0 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix},$$

we check that

$$\det \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} = -1 \neq 0.$$

So H exists and

$$H'(0, 1) = -\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} -1 & -\pi/2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} -1 & -\pi/2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & -1 + \pi/2 \\ -1 & -\pi/2 \end{bmatrix}.$$