Mathematical analysis 2, WNE, 2018/2019
meeting 20. – solutions

9 May 2019

1. Check the theorem of inverse function for $f(x,y)=(xy^2,x)$ and points $(1,1)$ and $(t,0)$, $t\in\mathbb{R}$.

   Generally there is no inverse function – it is not possible to get out of (xy^2,x) uniquely (x,y), because even though x can be determined uniquely, we do not know the sign of y. We can write that $(a,b)\mapsto(b,\pm\sqrt{a/b})$, but we cannot determine the sign.

   But in a neighborhood of a given point it may be possible. E.g. consider the point $(1,1)$. In its neighborhood $y>0$, so for $B((1,1),1/2)$, we get $f^{-1}(a,b)=(b,\sqrt{a/b})$ – and it works. Looking at the Theorem, indeed,

   $$f'=\begin{bmatrix} y^2 & 2xy \\ 1 & 0 \end{bmatrix},$$

   so

   $$f'(1,1)=\begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$$

   and $\det f'(1,1)=-2\neq 0$. Moreover, indeed

   $$(f')^{-1}=\begin{bmatrix} 0 & 1 \\ 1/2xy & -y/2x \end{bmatrix}$$

   is the same as

   $$(f^{-1}(a,b))'=\left(b,\sqrt{a/b}\right)'=\begin{bmatrix} 0 & 1 \\ 1/2xy & -y/2x \end{bmatrix}.$$

   On the other hand at $(x,0)$ (for any x) there is not inverse function since

   $$f'(x,0)=\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

   and so $\det f'(x,0)=0$.