Mathematical analysis 2, WNE, 2018/2019
meeting 19.

7 May 2019

Problems

1. Find the equation of the plane tangent to the surface described by the following equations at the indicated point.
   a) $\sqrt[3]{x} + \sqrt[3]{y} + \sqrt[3]{z} = 1$, $P = (1, -1, 1)$,
   b) $xyz + x^2 - 3y^2 + z^3 = 14$, $P = (5, -2, 3)$.
2. Find all points on the surface described $z = -x^2 - y^2 + 8x - 6y + 10$, at which the tangent plane is horizontal.
3. Determine all the points on the surface described by the equation
$$z = \frac{3}{4}y^2 + \frac{1}{24}y^3 - \frac{1}{32}y^4 - x^2,$$
   where the tangent plane to the surface is horizontal. Which of these points are local extrema of the function $z(x, y)$?
4. Find a diffeomorphism between the following pairs of domains:
   a) the interior of the triangle with vertices $(0, 0)$, $(0, 1)$ and $(1, 0)$ the interior of the triangle with vertices $(0, 0)$, $(0, 1)$ and $(2, 0)$,
   b) the interior of the triangle with vertices $(0, 0)$, $(0, 1)$ i $(1, 0)$ and the interior of the square with vertices $(0, 0)$, $(0, 1)$, $(1, 1)$ i $(1, 0)$,
   c) the interior of the triangle with vertices $(0, 0)$, $(0, 1)$ i $(1, 0)$ and the interior of the unit circle with centre $(0, 0)$,
   d) the interior of the triangle with vertices $(0, 0)$, $(0, 1)$ i $(1, 0)$ and the whole plane $\mathbb{R}^2$.
5. Find a diffeomorphism $f\colon A \to B$, where
$$A = \{(x, y) \in \mathbb{R}^2 \colon 1 < x^2 + y^2 < 4\},$$
$$B = \{(x, y) \in \mathbb{R}^2 \colon 1 < x^2 + y^2 < 9\}.$$

We will be having a short test at the beginning of our next meeting!