

# Mathematical analysis 2, WNE, 2018/2019 meeting 18.

30 April 2019

## Problems

1. Give an example of continuous functions of two variables, which has two local maxima, but no other local extrema.
2. Show that the function  $2(1 - e^{2y} + x^2)^3 - 3(1 - e^{2y} + x^2)^2 - 24x^2e^{2y}$  has exactly one critical point at which the function has a strict local maximum, but the function is neither bounded above or below.
3. Show that there is no function  $f(x, y)$  of  $C^2$  class such that  $\frac{\partial f}{\partial x}(x, y) = 6xy^2$  and  $\frac{\partial f}{\partial y}(x, y) = 8x^2y$ .
4. Determine if the following functions satisfies Laplace's equation:

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0.$$

- a)  $f(x, y) = \sqrt{x^2 + y^2}$ ,
- b)  $f(x, y) = \ln(\sqrt{x^2 + y^2})$ ,
- c)  $f(x, y) = e^{-x} \sin y$ .

5. Find and classify all the critical points of the following functions:

- a)  $f(x, y) = e^{xy} - 2xy$ ,
- b)  $f(x, y, z) = x^2 + y^2 + z^2 - xy + x + 2z$ .

## Homework

### Group 8:00

Find and classify all the critical points of the function  $f(x, y) = (2x^2 + y^2)e^{-x^2 - y^2}$ .

### Group 9:45

Find and classify all the critical points of the function  $f(x, y) = (x^2 + 2y^2)e^{-x^2 - y^2}$ .