Mathematical analysis 2, WNE, 2018/2019
meeting 18. – homework solutions

30 April 2019

Group 8:00

Find and classify all the critical points of the function $f(x,y) = (2x^2+y^2)e^{-x^2-y^2}$.

$$\frac{\partial f}{\partial x} = -2xe^{-x^2-y^2}(2x^2+y^2-2),$$
$$\frac{\partial f}{\partial y} = -2ye^{-x^2-y^2}(2x^2+y^2-1),$$

Are zero if either:

(1) $x = y = 0$,

(2) $x = 0$ and $y^2 = 1$, so $(0,1)$, $(0,-1)$,

(3) $y = 0$ and $x^2 = 1$, so $(1,0)$, $(-1,0)$,

(4) $2x^2+y^2-2 = 0$ and $2x^2+y^2-1 = 0$ – a contradiction.

We get 5 critical points $(0,0),(1,0),(-1,0),(0,1),(0,-1)$.
Second order partial derivatives:

$$\frac{\partial^2 f}{\partial x^2} = e^{-x^2-y^2}(8x^4+4x^2(y^2-5)-2y^2+4),$$
$$\frac{\partial^2 f}{\partial x \partial y} = 4xye^{-x^2-y^2}(2x^2+y^2-3),$$
$$\frac{\partial^2 f}{\partial y^2} = e^{-x^2-y^2}(x^2(8y^2-4)+4y^4-10y^2+2),$$

At $(0,0)$ we get the following Hessian:

$$\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$

is positive definite, so it is a minimum.
At $(1,0)$ we get the following Hessian:

$$\begin{bmatrix} -8/e & 0 \\ 0 & -2/e \end{bmatrix}$$

is negative definite, so it is a maximum.
At $(-1,0)$ we get the following Hessian:

$$\begin{bmatrix} -8/e & 0 \\ 0 & -2/e \end{bmatrix}$$

is negative definite, so it is a maximum.
At $(0,1)$ we get the following Hessian:

$$\begin{bmatrix} 2/e & 0 \\ 0 & -4/e \end{bmatrix}$$

is non-definite, so it is not a local extremum.

At $(0,-1)$ we get the following Hessian:

$$\begin{bmatrix} 2/e & 0 \\ 0 & -4/e \end{bmatrix}$$

is non-definite, so it is not a local extremum.

Group 9:45

Find and classify all the critical points of the function $f(x,y) = (x^2+2y^2)e^{-x^2-y^2}$.

$$\frac{\partial f}{\partial x} = -2xe^{-x^2-y^2}(x^2+2y^2-1),$$
$$\frac{\partial f}{\partial y} = -2ye^{-x^2-y^2}(x^2+2y^2-2),$$

Are zero if either:

(1) $x = y = 0$,

(2) $x = 0$ and $y^2 = 1$, so $(0,1)$, $(0,-1)$,

(3) $y = 0$ and $x^2 = 1$, so $(1,0)$, $(-1,0)$,

(4) $x^2+2y^2-1 = 0$ and $x^2+2y^2-1 = 0$ – a contradiction.

We get 5 critical points $(0,0),(1,0),(-1,0),(0,1),(0,-1)$.
Second order partial derivatives:

$$\frac{\partial^2 f}{\partial x^2} = e^{-x^2-y^2}(y^2(8x^2-4)+4x^4-10x^2+2),$$
$$\frac{\partial^2 f}{\partial x \partial y} = 4xye^{-x^2-y^2}(2y^2+x^2-3),$$
$$\frac{\partial^2 f}{\partial y^2} = e^{-x^2-y^2}(8y^4+4y^2(x^2-5)-2x^2+4).$$

At $(0,0)$ we get the following Hessian:

$$\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$$

is positive definite, so it is a minimum.
At $(0,1)$ we get the following Hessian:

$$\begin{bmatrix} -2/e & 0 \\ 0 & -8/e \end{bmatrix}$$

is negative definite, so it is a maximum.
At $(0,-1)$ we get the following Hessian:

$$\begin{bmatrix} -2/e & 0 \\ 0 & -8/e \end{bmatrix}$$

is negative definite, so it is a maximum.
At $(1,0)$ we get the following Hessian:

$$\begin{bmatrix} -4/e & 0 \\ 0 & 2/e \end{bmatrix}$$

is non-definite, so it is not a local extremum.
At $(-1,0)$ we get the following Hessian:

$$\begin{bmatrix} -4/e & 0 \\ 0 & 2/e \end{bmatrix}$$

is non-definite, so it is not a local extremum.