

Mathematical analysis 2, WNE, 2018/2019 meeting 17.

25 April 2019

Problems

1. Find and classify the local extrema of the following functions:

a) $f(x, y) = 3x^2 + 6xy + 2y^3 + 12x - 24y$,

b) $f(x, y) = x^3y - 3xy^2$.

2. Show that the function $f(x, y) = x^2(1 + y)^3 + y^2$ has exactly one critical point p . Show that p is a local minimum and $\sup_{(x,y) \in \mathbb{R}^2} f(x, y) = +\infty$ and $\inf_{(x,y) \in \mathbb{R}^2} f(x, y) = -\infty$.

3. The P , V and T denote pressure P , volume V , and temperature T of a given gas, which satisfies the equation $PV = RT$, where R is a certain constant. Prove that

$$\frac{\partial P}{\partial V} \cdot \frac{\partial V}{\partial T} \cdot \frac{\partial T}{\partial P} = -1.$$

4. Find the maximal and minimum value of

$$f(x, y) = \sin x + \sin y + \sin(x + y)$$

on the set $D = \{(x, y) \in \mathbb{R}^2 : 0 \leq x, y \leq \pi/2\}$.

5. Among triangles inscribed inside a circle of radius R find the one with maximal area.

6. Among rectangular boxes inscribed inside a sphere of radius R find the one with maximal volume.

Homework

Group 8:00

Find and classify the local extrema of $f(x, y) = x^3 - 3xy^2$.

Group 9:45

Find and classify the local extrema of $f(x, y) = (x - y)(xy - 1)$.