Mathematical analysis 2, WNE, 2018/2019
meeting 17. – homework solutions

25 April 2019

Group 8:00

Find and classify the local extrema of $f(x,y) = x^3 - 3xy^2$.

$$\frac{\partial f}{\partial x} = 3x^2 - 3y^2,$$
$$\frac{\partial f}{\partial y} = -6xy,$$

are zero if $x = y$, thus $x = y = 0$, and the only critical point is $(0,0)$.
It is not an extremum, since for $x = 0$ the function is constant and equal to 0.

Group 9:45

Find and classify the local extrema of $f(x,y) = (x-y)(xy-1)$.

$$\frac{\partial f}{\partial x} = xy - 1 + (x-y)y = 2xy - 1 - y^2,$$
$$\frac{\partial f}{\partial y} = -(xy-1) + (x-y)x = -2xy + 1 + x^2,$$

We get two critical points $(1,1)$ and $(-1,-1)$.
Second order partial derivatives are

$$\frac{\partial^2 f}{\partial x^2} = 2y,$$
$$\frac{\partial^2 f}{\partial x \partial y} = 2x - 2y,$$
$$\frac{\partial^2 f}{\partial y^2} = -2x,$$

So Hessian and $(1,1)$ takes form

$$\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$$

which is non-definite since $d^2(h_x, h_y) = 2h_x^2 - 2h_y^2$ gives value > 0 e.g. for $h_x = 1, h_y = 0$ and value < 0 for $h_x = 0, h_y = 1$. So it is not an extremum.
Hessian at $(-1,-1)$ is

$$\begin{bmatrix} -2 & 0 \\ 0 & 2 \end{bmatrix}$$

and by Sylvester's criterion ($\det A_1 = -2$, $\det A_2 = -4$) it is non-definite and thus is not an extremum.