Mathematical analysis 2, WNE, 2018/2019
meeting 16.

16 April 2019

Problems

1. Let
$$f(x,y) = \begin{cases} \dfrac{xy(x^2-y^2)}{x^2+y^2} & \text{, for } (x,y) \neq (0,0), \\ 0 & \text{, for } (x,y) = (0,0). \end{cases}$$
Show that
   a) the point $(0,0)$ is a critical point of the function,
   b) all second order partial derivatives $\frac{\partial^2 f}{\partial x^2}$, $\frac{\partial^2 f}{\partial y^2}$, $\frac{\partial^2 f}{\partial x \partial y}$ and $\frac{\partial^2 f}{\partial y \partial x}$ exist at $(0,0)$, but
$$\frac{\partial^2 f}{\partial x \partial y}(0,0) \neq \frac{\partial^2 f}{\partial y \partial x}(0,0).$$
   c) the point $(0,0)$ is not a local extremum of f.
2. Let $f(x,y) = (y-x^2)(y-3x^2)$. Show that
   a) $f'(0,0) = (0,0)$,
   b) for every $(a,b) \in \mathbb{R}^2 \setminus \{(0,0)\}$, the function $h(t) = f(ta,tb)$ has a local minimum for $t = 0$,
   c) f does not have a local minimum at $(0,0)$.
3. Let $A = \{(x,y,z) \in \mathbb{R}^2 \colon 2x - 3y + z = 1\}$. Find a point $p \in A$ closest to $(3,-2,1)$.
4. Find the maximum possible volume of a cylinder whose height plus circumference of the base does not exceed 108cm.
5. Find and classify the local extrema of the following functions:
   a) $f(x,y) = x^3 + y^3 + 3xy + 3$,
   b) $f(x,y) = e^{-x^4-y^4}$.

Homework

Group 8:00

Find the maximum volume of the parallelepiped for which the sum of all three sides (length, width, height) does not exceeds 108 cm.

Group 9:45

Find the maximum volume of the parallelepiped for which the sum of all three sides (length, width, height) does not exceeds 54 cm.