Mathematical analysis 2, WNE, 2018/2019
meeting 16. – homework solutions

16 April 2019

Group 8:00

Find the maximum volume of the parallelepiped for which the sum of all three sides (length, width, height) does not exceeds 108 cm.

It is clear that out of parallelepipeds the best one is a cuboid (indeed, the volume is the surface area of the base times the height, and the height equals the length of a side only then). We get $z = 108 - x - y$, so $V(x, y) = xy(108 - x - y)$, thus the partial derivatives are

$$(108 - x - y)y - xy$$

and

$$(108 - x - y)x - xy,$$

and since $x = 0$ or $y = 0$ is not in the domain, we ask when $x = 108 - x - y$, i.e. $x = (108 - y)/2$, so $108 - (108 - y)/2 = 2y$, thus $54 = 3y/2$, $y = 36 = x = z$, and therefore the maximal volume is $36^3 = 46656$ cm^3.

Group 9:45

Find the maximum volume of the parallelepiped for which the sum of all three sides (length, width, height) does not exceeds 54 cm.

It is clear that out of parallelepipeds the best one is a cuboid (indeed, the volume is the surface area of the base times the height, and the height equals the length of a side only then). We get $z = 54 - x - y$, so $V(x, y) = xy(54 - x - y)$, thus the partial derivatives are

$$(54 - x - y)y - xy$$

and

$$(54 - x - y)x - xy,$$

and since $x = 0$ or $y = 0$ is not in the domain, we ask when $x = 54 - x - y$, i.e. $x = (54 - y)/2$, so $54 - (54 - y)/2 = 2y$, thus $27 = 3y/2$, $y = 18 = x = z$, and therefore the maximal volume is $18^3 = 5832$ cm^3.