Mathematical analysis 2, WNE, 2018/2019
meeting 9.

19 March 2019

Problems

1. Check whether the limit exists. If it does find it.

   a) $\lim_{(x,y)\to(0,0)} \dfrac{x^2-y^2}{\sqrt{x^2+y^2}}$,

   b) $\lim_{(x,y)\to(0,0)} \dfrac{x^2-y^2}{x^2+y^2}$,

   c) $\lim_{(x,y,z)\to(0,0,0)} \dfrac{xy+yz+zx}{x^2+y^2+z^2}$.

2. Determine whether the functions are continuous.

   a) $f(x,y) = \begin{cases} \dfrac{x-y}{x^3-y} & \text{, for } y \neq x^3 \\ 1 & \text{, for } y = x^3, \end{cases}$

   b) $f(x,y) = \begin{cases} \dfrac{x^3+y^3}{x^2+y^2} & \text{, for } (x,y) \neq (0,0) \\ 0 & \text{, for } (x,y) = (0,0), \end{cases}$

   c) $f(x,y) = \begin{cases} \dfrac{2xy^3+x^2y^3}{x^4+2y^4} & \text{, for } (x,y) \neq (0,0) \\ 0 & \text{, for } (x,y) = (0,0), \end{cases}$

3. Prove that the function

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & \text{, for } (x,y) \neq (0,0) \\ 0 & \text{, for } (x,y) = (0,0), \end{cases}$$

   is not continuous at $(0,0)$.

We will be having a short test at the beginning of our next meeting!