# Mathematical analysis 2, WNE, 2018/2019
# meeting 6.

7 March 2019

## Problems

1. Prove that every norm generated by an inner product satisfies (the parallelogram law)
$$\|u+v\|^2 + \|u-v\|^2 = 2(\|u\|^2 + \|v\|^2),$$
for every points $u, v \in \mathbb{R}^n$.
2. Prove that every inner product $\langle \cdot, \cdot \rangle$ and the norm $\| \cdot \|$ the norm generated by it satisfy the following condition:
$$\langle u, v \rangle = \frac{1}{4}(\|u+v\|^2 - \|u-v\|^2),$$
for every points $u, v \in \mathbb{R}^n$.
3. Prove the Jordan-von Neumann Theorem, which states that every norm satisfying the parallelogram law is generated by an inner product (hint: the previous problem).
4. Prove that the unit ball for every norm is convex.
5. Let $W \subseteq \mathbb{R}^n$ be a convex set such that:
   a) for every $v \in \mathbb{R}^n$, there exists $t \in \mathbb{R}$, such that $v \in tW = \{tw \colon w \in W\}$,
   b) for every $w \in W$ and $r \in [-1, 1]$, $rw \in W$,
   c) there exists $R > 0$, such that for every $(w_1, \ldots, w_n) \in W$, $w_1^2 + \ldots + w_n^2 \leqslant R$.

   Prove that
$$\|v\| = \inf\{t > 0 \colon tv \in W\}$$
   is a norm in $\mathbb{R}^n$.

## Homework

### Group 8:00

Consider a function $\|(x, y)\| = |xy|$. Which of the conditions from the definition of the norm are satisfied by this function? Which are not? Why?

### Group 9:45

Consider a function $\|(x, y)\| = |x + y|$. Which of the conditions from the definition of the norm are satisfied by this function? Which are not? Why?