Mathematical analysis 2, WNE, 2018/2019
meeting 12. – solutions

28 March 2019

1. What is the direction in which f grows most rapidly P?

   a) $f(x,y) = \sin\frac{\pi xy}{4}$, $P = (3,1)$,

   $$f' = \left[\frac{y\pi\cos\pi xy/4}{4}, \frac{x\pi\cos\pi xy/4}{4}\right]$$
   $$f'(3,1) = \left[\frac{\pi\cos\pi 3/4}{4}, \frac{3\pi\cos\pi 3/4}{4}\right] = \left[\frac{\pi\sqrt{2}}{8}, \frac{3\pi\sqrt{2}}{8}\right]$$

   and this is the direction which we are looking for.

   b) $f(x,y,z) = e^x\sin y + e^y\sin z + e^z\sin x$, $P = (0,0,0)$.

   $$f' = [e^x\sin y + e^z\cos x, e^x\cos y + e^y\sin z, e^y\cos z + e^z\sin x]$$
   $$f'(0,0,0) = [1,1,1]$$

   and this is the direction which we are looking for.

2. Assume that we are at point $(-100,-100,430)$ on a mountain described by $z = 500 - 0.003x^2 - 0.004y^2$.

   a) In which direction the slope of the mountain is steepest?

   $$z' = [-0.006x, -0.008y]$$
   $$z'(-100,-100) = [0.6, 0.8]$$

   b) How steep is the mountain at this point (calculate the angle of the steepest tangent line)?

   $$\|(0.6,0.8)\| = \sqrt{0.36+0.64} = 1 = \operatorname{tg}\alpha,$$

   so $\alpha = \pi/4$.

3. Find an equation of the tangent line to the curve described by the equation at point P?

   a) $2x^3 + 2y^3 - 9xy = 0$, $P = (1,2)$,

   $$h' = [6x^2 - 9y, 6y^2 - 9x]$$
   $$h'(1,2) = [6-18, 24-9] = [-12, 15]$$

   Thus $T(L)$: $-12x + 15y = 0$, so $L : -12x + 15y = 18$.

   b) $x^4 + xy + y^2 = 19$, $P = (2,-3)$.

   $$h' = [4x^2 + y, x + 2y]$$
   $$h'(2,-3) = [32-3, 2-6] = [29, -4]$$

   Thus $T(L)$: $29x - 4y = 0$, so $L : 29x - 4y = 70$.